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56 MHZ Cavity Prototype Measurements

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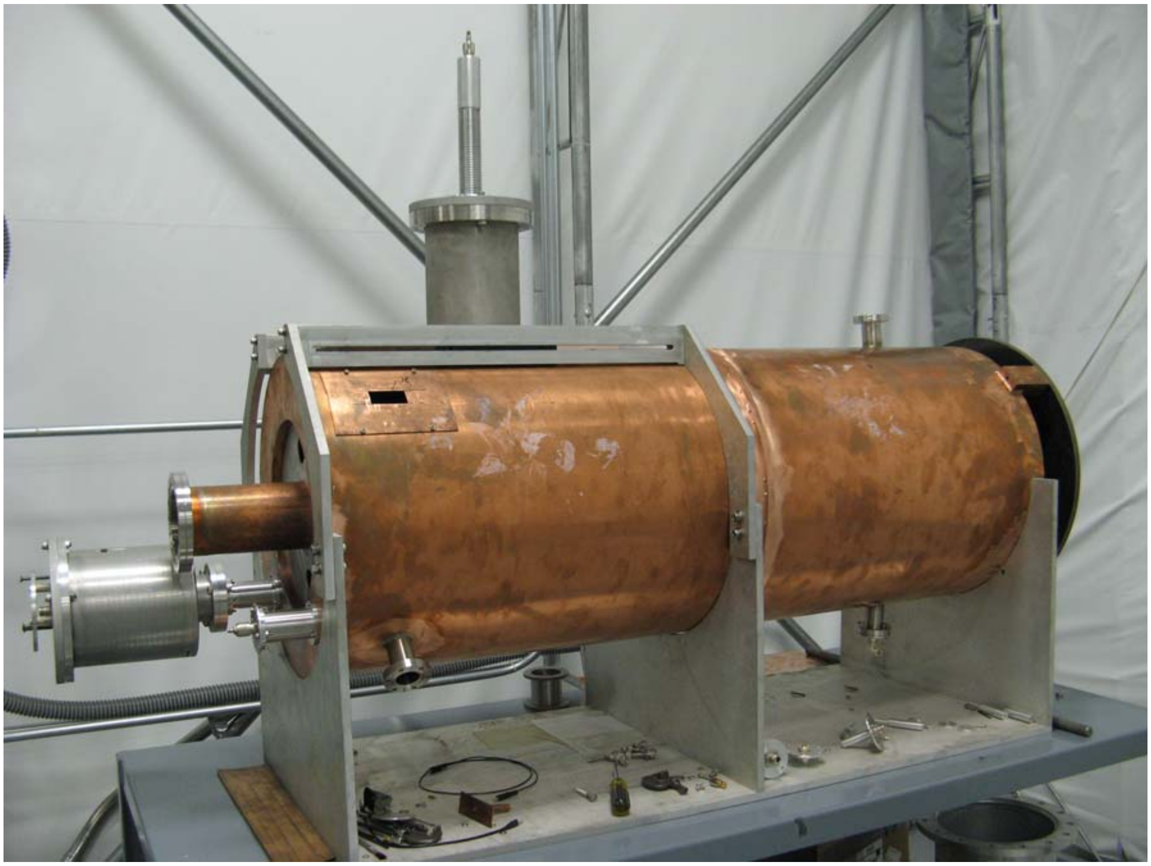


FIGURE 1: 56 MHz Cavity Cu Model with Fundamental and HOM Coupler

I. MEASUREMENTS PERFORMED

The following series of measurements were performed on the Cu model in order to determine the physics requirements and to demonstrate feasibility.

Slow tuner frequency change

Fundamental Damper

External Q as function of position
Location of position for minimum frequency change

HOM Damper

External Q at 56 MHz with single damper
Q values at critical harmonics ( ) with single damper

Pick-up probe

Capacitive at acceleration gap

Magnetic at shorted end (postponed for SC measurement)

Self inductance of fundamental damper loop

II. INTERPRETATION of MEASUREMENTS

All measurements involved the use of a network analyzer. The well known difference between accelerator parameters and circuit parameters, provided by the network analyzer, can cause confusion. At first the frequently used equivalent circuit parameters with their definition is given.

Equivalent Circuit Analysis

SuperFish + Microwave Study provide

- stored energy U_C and the power lost in the cavity P_C at accelerating field (index C as in Padamsee and index CD for circuit definition)

$$V_C = V_{CD}\sqrt{2}$$

- shunt impedance (index A for accelerator definition and SH shunt impedance in circuit definition)

$$R_A = V_C^2 / P_C \text{ and}$$

$$R_{SH} = V_{CD}^2 / P_C = R_A / 2$$

- intrinsic quality factor $Q_0 = \dfrac{\omega_0 U_C}{P_C}$ yielding the cavity inductance

$$L = \frac{1}{2\omega_0}\left(\frac{R_A}{Q_0}\right)$$

In order to ovoid errors and to simplify the interpretation of measurements, a list of typical measurements is given here.

The port coupler strength

The coupler strength characterized in terms of β or transformer ratio $n:1$ is obtained from an 21 measurement which yields the impedance

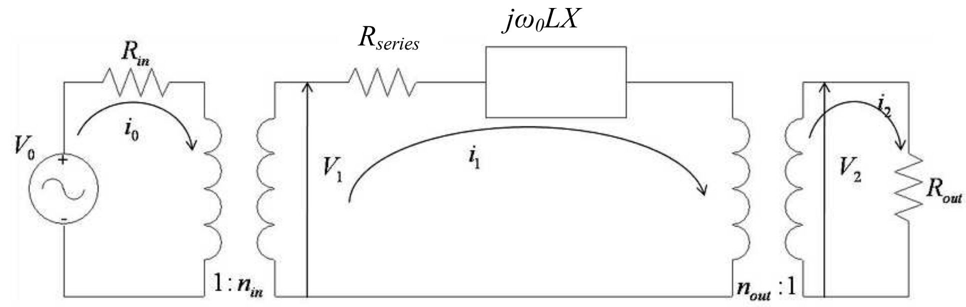
$$R_{Port} = R_0 \frac{1-S_{21}}{1+S_{21}} \text{ with } R_0 = 50\ \Omega$$

from which follows

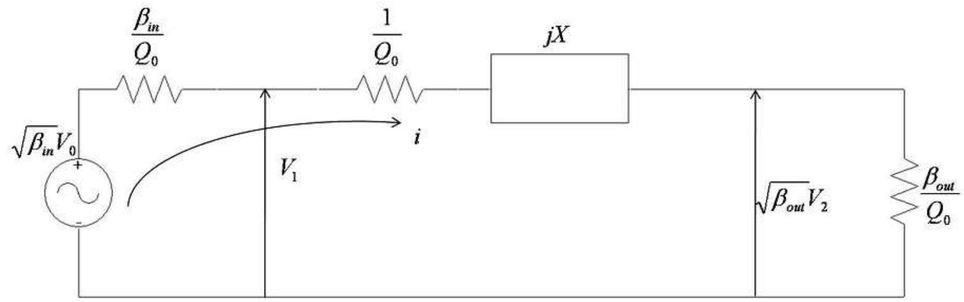
$$\beta = \frac{R_{Port}}{R_{SH}} \text{ and } n = \sqrt{\frac{R_{SH}}{R_{Port}}}$$

Note that once measured at room temperature n remains constant during cool-down. The accuracy from the S21 measurements is reduced for a weak coupler such as a pickup probe and must by replaced by the following measurement of the Q-external .

Measurement of Q-external



$$S_{21} = \frac{2V_2}{V_0} = \frac{2n_{in}n_{out}R_{out}}{R_{Series} + n_i^2 R_{in} + n_o^2 R_{out} + j\omega_0 L\Delta}$$

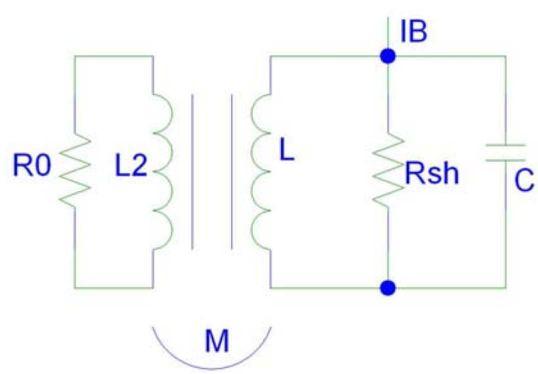


$$S_{21} = \frac{2\sqrt{\beta_{in}\beta_{out}}}{1 + \beta_{in} + \beta_{out} + Q_0\Delta} = \frac{2Q_L}{\sqrt{Q_{in}Q_{out}}\left(1 + jQ_L\Delta\right)}$$

Convenient method to find Q-external ($Q_{ext} = Q_{out}$) involves critically coupled $Q_{in} = Q_0$
Yielding at resonance, with $\Delta = 0$ and $Q_L = Q_0 / 2$,

$$Q_{ext} = Q_0 / S_{21}^2$$

The dissipated power to the (external) load



Ignoring the bunch length, the time averaged beam current I_B produces a cavity voltage

$$V_{CD} = Z_{CD}\frac{2I_B}{\sqrt{2}}$$

The generic cavity impedance with L, C, R_T in circuit definition is

$$Z_{CD} = \frac{\omega_0 L Q_L}{1 + jQ_L\Delta} = \frac{R_L}{1 + jQ_L\Delta} \quad \text{with } \Delta = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

$$\frac{1}{R_L} = \frac{1}{R_{SH}} + \frac{1}{R_X}$$

The total power into the cavity

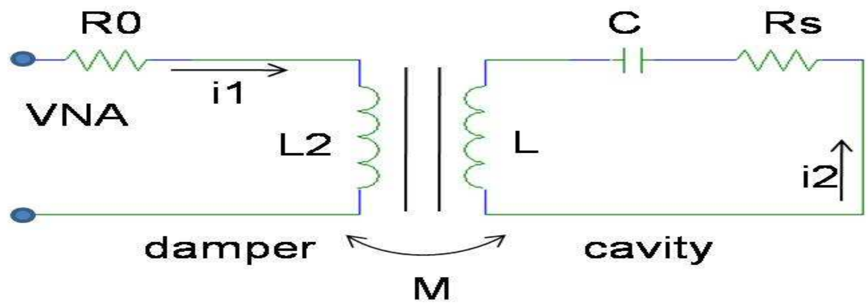
$$P_T = 2\frac{Z_{CD}^* Z_{CD}}{R_L} I_B^2 = 2\frac{R_{SH}}{Q_0}\frac{Q_L}{1 + (Q_L\Delta)^2} I_B^2$$

The external power

$$P_X = \frac{Q_L}{Q_X} P_L = 2\frac{R_{SH}}{Q_X}\frac{Q_L^2}{1 + (Q_L\Delta)^2} I_B^2$$

Cavity Mutual inductance *M* and Loop Self Inductance *L₂*

Determining the self inductance of the loops and mutual inductance of the FPC and the HOM coupler presented a specially difficult task not easily solved by theory.
M and L_2 were obtained from a S11 measurement and its application to the following circuit



The Vector Network Analyzer provides input impedance

$$Z = R_0 \frac{1 - S_{11}}{1 + S_{11}}$$

which is follows from the circuit as

$$Z = j\omega L_2 + \frac{(\omega M)^2}{R_S + j\omega_0 L\Delta} \quad \text{with } \Delta = \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

In terms of the series resistor $R_S = \omega_0 L / Q_0$ and $L = \frac{1}{2\omega_0}\left(\frac{R_A}{Q_0}\right)$ already as defined above

One finds the expressions for the real part and imaginary part as follows,

$$\operatorname{Re} Z = \frac{(\omega M)^2}{R_S^2 + (\omega_0 L\Delta)^2}$$

$$\operatorname{Im} Z = \omega L_2 - \frac{\omega_0 L\Delta(\omega M)^2}{R_S^2 + (\omega_0 L\Delta)^2}$$

At resonance , $\operatorname{Re} Z$ will give the mutual inductance M , and
$\operatorname{Im} Z$ will give the self inductance L_2 separately.
The results for the fundamental Damper are
$M \approx 11$ nH and $L_2 = 261$ nH, the latter being a surprisingly large value.

References

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2. E. M. Choi and H. Hahn, *Summary of the fundamental mode damper experiments of the 56 MHz SRF cavity*, BNL Report C-AD/AP/314 , July 2008
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