Effect of Beam Current Fluctuations in the Electron Cooler on the Cooled Ion Beam in RHIC II.

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Abstract

In this paper, I consider an effect of fluctuations of the electron beam current in the designed BNL electron cooler on the ion beam in RHIC II. Using these estimates, I show that the "heating" effect of the electron current noise on the cooled ion beam is negligible if the charge-per-bunch noise is kept below one per cent.

1 Introduction

The proposed BNL electron cooler will be used to cool ion beams in the RHIC II upgrade. If the charge of electron bunches fluctuates, the electron beam current noise can produce a "heating" effect on the cooled ion beam. Without discussing sources of fluctuations of the charge of electron bunches, I estimate the effect of the electron beam intensity noise in a linear approximation assuming a broad spectrum of the current noise.

2 Focusing by electron bunches

If the effect of cooling is neglected, the interaction of ions with the electron beam can be described, in the linear approximation, as additional focusing. To estimate the focusing effect one can assume that the transverse displacement of ions does not change when ions travel through the cooling section. This assumption can be justified because the betatron phase advance in the cooling section is of the order of 0.25 radian ($L/\beta = 100m/400m = 0.25$) and can be neglected. Additionally, as will be shown later, the focusing length of the ion-electron interaction is much larger than the length of the cooling section. Therefore, the interaction of RHIC ions with the electron beam can be considered as a thin focusing lens.

Assuming that electron bunches have a uniform longitudinal distribution of length l and an axially symmetric transverse distribution, the linear part of the transverse focusing force is given by (in CGS units)

$$F_{\perp} = -k\frac{ZeQ}{\gamma^2\sigma^2 l}x \tag{1}$$

where Z is the charge state of RHIC ions, Q is the charge of electron bunches, e is the elementary charge, γ is the relativistic factor, σ is the transverse r.m.s. size of electron bunches, and x is the displacement from the center of the electron beam trajectory. k is a numerical factor depending on the shape of the transverse distribution and is equal to

$$k = \frac{2}{3}\,(linear)$$
$$k = 1\,(gaussian)$$

for linear and Gaussian distributions respectively.

The deflection angle produced by the force (1) acting on a relativistic ion over the distance L_c is given by

$$x' = \frac{F_\perp L_c}{pc} = -k\frac{N_e Z r_p L_c}{\gamma^3\sigma^2 Al}x, \quad (2)$$

where the p is the momentum on ions, c is the speed of light, r_p is the classical proton radius, N_e is the number of electrons in an electron bunch, and A is the atomic weight of ions. Thus, the inverse focusing length of the effective lens is given by

$$d = \frac{1}{f} = -k\frac{r_p N_e Z L_c}{\gamma^3\sigma^2 Al}. \quad (3)$$

Beam parameters that will be delivered by the electron cooler are given in Table 1.

Table 1: Parameters of the electron beam delivered by the BNL cooler

Parameter	Value
γ	107
Charge-per-bunch	5 nC
Emittance (norm.)	5 μm
β-function (cool. section)	400 m
Bunch length (Uniform dist.)	3.4 cm

For the parameters shown in Table 1, fully-stripped gold ions, and a uniform transverse distribution of electron bunches, the focusing length of the effective electron lens is approximately equal to $6 \cdot 10^5$ m. This focusing length is much larger than the length of the cooling section that, as discussed above, justifies the impulse approximation of the interaction of RHIC ions with the electron beam.

3 Amplitude growth induced by charge-per-bunch noise with a wide spectrum

The focusing produced by the electron beam depends linearly on the charge of electron bunches. Therefore, the additional focusing δK due to fluctuations of the charge of electron bunches can be added to the equation of motion as follows:

$$x'' + K(s)x = -\delta K(s)x \quad (4)$$

with the periodic focusing $K(s)$ and the extra, non-periodic, term

$$\delta K = \sum_n d_n \delta(s - nC) \quad (5)$$

where d_n is the extra focusing strength of the $n-th$ interaction due to the variation of the number of electron and $\delta(s - nC)$ is the periodic delta-function with the period equal to the RHIC circumference C.

Using the displacement normalized to the absolute value of the Floquet function w, which is related to the β-function simply as $w = \sqrt{\beta}$,

$$y = \frac{x}{w} \quad (6)$$

and the betatron phase ψ as an independent variable instead of s, one can rewrite equation (4) as

$$\ddot{y} + \nu^2 y = -\nu^2 w^4 \delta K(s) y, \quad (7)$$

where the dots mean differentiation with respect to the betatron phase $d/d\psi$ and ν is the betatron frequency in RHIC. To clarify the introduced parameterization note that the solution of the homogeneous equation of motion is given in the new variables by

$$x = aw\cos{(\nu\psi + \phi)}, \tag{8}$$

where a and ϕ are constants.

As equation (7) suggests, the motion of ions outside the cooling section is identical to the equation of motion of a free oscillator. The integral of motion of this equation, which is proportional to the squared amplitude of oscillations, can be found by a trivial integration and is equal to:

$$\dot{y}^2 + \nu^2 y^2 = const = \nu^2 a^2. \tag{9}$$

Because we neglected the phase advance in the cooling section, the additional focusing due to a random fluctuation of the charge of electron bunches induces only a deflection angle $\delta\dot{y}$. This angle variation causes the amplitude of oscillations to change according to

$$(\dot{y} + \delta\dot{y})^2 + \nu^2 y^2 = \nu^2(a^2 + \delta a^2), \tag{10}$$

yielding the variation of the square of the amplitude

$$2\dot{y}\delta\dot{y} + (\delta\dot{y})^2 = \nu^2\delta a^2. \tag{11}$$

A temporal behavior of the amplitude of oscillations, a, depends on specifics of the electron current noise spectrum. As mentioned before, it is assumed in this paper that the noise spectrum is much wider than the revolution frequency. In other words, the correlation time is assumed to be shorter than the revolution period and the additional focusing "kicks" experienced by an ion on different turns are uncorrelated. Therefore, averaging over time, which is equivalent to averaging over all possible variations of the electron bunch charge, yields the average variation of the amplitude a in a single collision:

$$\langle\delta a^2\rangle = \frac{\langle(\delta\dot{y})^2\rangle}{\nu^2}. \tag{12}$$

Assuming that the amplitude changes slowly comparatively to the revolution period T, the averaged time derivative of a can be expressed as

$$\langle\frac{da^2}{dt}\rangle = \frac{\langle\delta a^2\rangle}{T} = \frac{\langle(\delta\dot{y})^2\rangle}{\nu^2 T}. \tag{13}$$

Expressing the amplitude growth via x' as $\dot{y} = \nu w_c \delta x'$, one gets

$$\langle\frac{da^2}{dt}\rangle = \frac{\langle(\delta x')^2\rangle w_c^2}{T}, \tag{14}$$

where w_c is the absolute value of the Floquet function at the cooling section. Note that the last formula yields the average growth rate of the betatron amplitude. At any given moment, the amplitude can either grow or decay because of the random nature of the process.

The angle $\delta x'$ produced by the additional focusing due to the charge-per-bunch ripple is $\delta x' = \delta d \cdot x$. The mean square of $\delta x'$ is

$$\langle(\delta x')^2\rangle = \langle(\delta d)^2\rangle \cdot \langle x^2\rangle, \tag{15}$$

where $\langle x^2\rangle$ is the mean square of the ion displacement at the cooling section. Because the beam displacement is expressed via the betatron amplitude and phase as

$$x = aw\cos(\nu\psi + \phi), \tag{16}$$

the mean square value of the ion displacement at the cooling section is given by

$$\langle x^2 \rangle = \frac{w_c^2 a^2}{2}. \tag{17}$$

Plugging (15) and (17) into (14) yields the equation for the average growth rate of the betatron amplitude:

$$\langle \frac{da^2}{dt} \rangle = \frac{\beta_c^2 d^2 a^2}{2T} \cdot \frac{\langle (\delta d)^2 \rangle}{d^2}, \tag{18}$$

where d is the focusing strength of electron bunches with the nominal charge-per-bunch, given by (3) and β_c was substituted for w_c^2. The last equation shows that the amplitude of oscillations of an ion grows, in average, exponentially with the growth time given by:

$$\tau = \frac{4T}{\beta_c d^2} \frac{1}{\frac{\langle (\delta d)^2 \rangle}{d^2}}. \tag{19}$$

Note that the amplitude growth time does not depend on initial conditions and therefore should be the same for all ions in a bunch. This indicates that the r.m.s. beam size will grow exponentially in average with the growth time given by (19).

4 Noise-driven parametric resonance(s) - not emittance growth

Although the derivation presented above can be interpreted as an indication of the beam emittance growth, the r.m.s. emittance actually does not change within the approximations made in the previous section. It is trivial to show that linear transformations with the determinant equal to unity do not change the r.m.s. emittance. Random variations of linear focusing do, however, drive parametric resonances causing beatings of the β-function with the amplitude increasing with time. The possibility of excitation of parametric resonances by randomly changing focusing can be demonstrated using the equation for small deviations of the Floquet function. In the linear approximation, the absolute value of the Floquet function can be written as $w + \delta w$, where δw is small variation from the equilibrium value of w. The equation for the normalized deviation $z = \delta w / w$ is

$$\ddot{z} + 4\nu^2 z = -\nu^2 w^4 \delta K. \tag{20}$$

The last equation can be solved using the method of variation of parameters. That is, the solution of (20) is written in the form $z = C(\psi)e^{2i\nu\psi} + c.c.$. The rate of change of the square of the absolute value of C is proportional to the spectral power at frequencies $\nu = k/2$, where k is an integer number:

$$\frac{d|C|^2}{d\psi} \propto \sum_{k=-\infty}^{\infty} S_d(2\nu - k) \rightarrow \int_{-\infty}^{\infty} S_d(2\nu - k)dk = R_d(\tau = 0) = \langle (\delta d)^2 \rangle. \tag{21}$$

This fact directly points at the excitation of parametric resonances. For a broad noise spectrum, the sum can be converted to an integral over the spectral density, which is equal to the correlation function at a zero shift time, which is, in turn by definition, equal to the mean square value of the focusing strength error.

5 Numerical example

Figure 1 shows the growth time given by equation (19) vs. the noise r.m.s. value for the electron beam parameters listed in Table 1 and fully-stripped gold ions. For the 1% r.m.s. electron charge-per-bunch noise, the exponential growth time is equal to $1.2 \cdot 10^6$ sec., that is, approximately 330 hours.

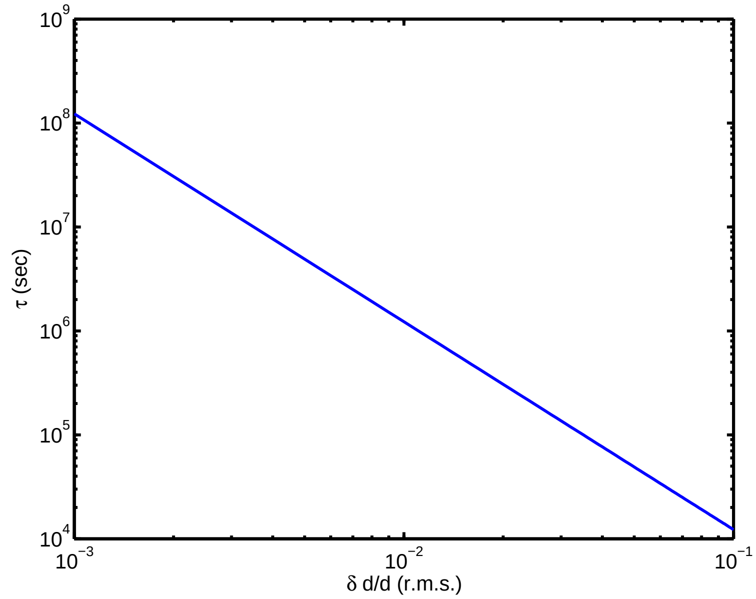

Figure 1: Growth time given by (19) vs. r.m.s. noise value.

6 Conclusions and Discussions

Fluctuations of the electron beam current in the electron cooler can drive parametric resonances and cause the size of the ion beam in RHIC II to grow. However, calculations presented above show that the focusing produced by the electron beam is weak due to the relativistic cancellation. Therefore, the electron current noise level has to be larger than one per cent to cause the ion beam size growth noticeable on the time scale of a single store (5-10 hours).

The presented analysis does not include non-linear effects and decoherence. Either one of those can cause real emittance growth instead of coherent beam envelope oscillations. Results of simulations for a Gaussian transverse profile of electron bunches that were not presented here indicate that the emittance growth due to nonlinearities should not present a significant problem if the noise is kept below 1%.