NSNS TRANSVERSE INSTABILITY

BNL/NSNS TECHNICAL NOTE

NO. 033

S. Y. Zhang

April 24, 1997

ALTERNATING GRADIENT SYNCHROTRON DEPARTMENT
BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973

NSNS Transverse Instability

1 Introduction

The NSNS storage ring transverse instabilities will be estimated in this report without considering Landau damping. The factors that are relevant to the transverse instabilities are the transverse impedance, the beam signal, the effective spectrum line, and the chromatic effect.

The detailed transverse impedance budget yet to be worked out. Also, careful impedance measurement is indispensable to finally address the machine impedance. Therefore, we use the impedance learned from the similar low energy proton synchrotrons in operation to estimate the NSNS performance.

The multiturn injection constitutes almost the entire cycle in the NSNS storage ring, the beam line densities will be considerably different from the extracted beam. In this report, however, we only consider the beam that is ready to extract.

The bunched beam instability criteria will be applied. Since the injection period is dominant, and also the bunching factor is large, the coasting beam instability criteria may well be applied.

2 Transverse Instability Formulation

Using the first orthogonal polynomial in the beam signal expansion, the bunched beam dynamic equation becomes a scale equation for the azimuthal mode m, [1],

$$\omega - \omega_\beta - m\omega_S = \frac{j\beta e I_0}{2Rm_0\gamma\nu_0\omega_0} \sum_{n=-\infty}^{\infty} Z_T(n) \left(\Lambda_0^{(m)}(n')\right)^2 \tag{2-1}$$

where ω_β and ω_S are the betatron and synchrotron frequencies, respectively, R is the average machine radius. The beam average current I_0 is defined by,

$$I_0 = \frac{Ne\omega_0}{2\pi} \tag{2-2}$$

where N is the number of particle, and ω_0 is the angular revolution frequency. Also $Z_T(n)$ is the transverse impedance, and $\Lambda_0^{(m)}(n)$ is the beam line density spectrum. n represents the spectrum lines in frequency domain that are effective. The notation n' denotes the beam spectrum frequency shift due to the chromatic effect.

For NSNS, the beam prior to extraction has a bunching factor $B_f \approx 0.4$. The beam line density spectra with $m = 0, 1, 2$ are shown in Fig.1, where for simplicity the Gaussian distribution is used. In this report, we consider only the first orthogonal polynomial in expanding the line density, i.e. $k = 1$.

Let us consider first the mode $m = 0$. The equation (2-1) becomes,

$$\omega - \omega_\beta = \frac{j\beta e I_0}{2Rm_0\gamma\nu_0\omega_0} \sum_{n=-\infty}^{\infty} Z_T(n)(\Lambda_0^{(0)}(n'))^2 \tag{2-3}$$

Note that the beam power spectrum $(\Lambda_0^{(0)}(n'))^2$ presented in (2-3) is always real. Thus, a positive real impedance implies that $\Delta\omega \sim j$. Substituting this into $e^{j\Delta\omega t}$, we find that the mode is stable. In a similar way, a negative real impedance implies instability, and imaginary impedance implies a frequency shift. The summation in (2-3) is because of the bunched beam, which has a discrete spectrum. For a coasting beam, a single spectrum line is effective due to the continuous wave, and the summation is eliminated.

In this report, the instabilities caused by the real impedances will be discussed. The imaginary impedance causes frequency shift, which as by-product will also be shown. The Landau damping will be the subject of another report.

3 Transverse Impedance

3.1 Resistive wall impedance

The transverse resistive wall impedance is defined as,

$$Z_T(\omega) = (sgn(\omega) + j)\frac{RZ_0\delta_s}{b^3} \tag{3-1}$$

where Z_0 is the impedance in free space, 377 Ω, b is the radius of the vacuum chamber, and δ_s is the skin depth at the frequency ω,

$$\delta_s = \sqrt{\frac{2\rho}{\mu_0|\omega|}} \tag{3-2}$$

where ρ is the resistivity of the vacuum chamber, for stainless steel it is $\rho = 0.73 \times 10^{-6}\ \Omega m$, and $\mu_0 = 4\pi \times 10^{-7}\ H/m$ is the permeability of free space.

For the NSNS [2], $R = 35.124\ m$, and the average beam pipe radius is taken as $b = 10$ cm. At the revolution frequency 1.189 MHz, we get $\delta_s = 0.394\ mm$, and the transverse resistive wall impedance at the revolution frequency is $Z_T(\omega_0) = (1+j)\ 5.22\ K\Omega/m$. Since the fractional tune is 0.82, the most damaging resistive wall impedance is $Z_T(-0.18\omega_0) = (-1+j)$ 12.31 $K\Omega/m$. The NSNS wall impedance is shown in Fig.2.

3.2 Space charge impedance

The conventional space charge impedance is defined as,

$$Z_T(\omega) = -j\frac{RZ_0}{\beta^2\gamma^2}(\frac{1}{a^2} - \frac{1}{b^2}) \tag{3-3}$$

where for Gaussian distribution, $a = \sqrt{2}\sigma$, with σ being the *rms* beam size. For NSNS, $\beta = 0.875$, $\gamma = 2.0658$, taking $a = \sqrt{2}\sigma = \sqrt{2} \times 1.67 = 2.36\ cm$, then the space charge impedance is $Z_T(\omega) = -j\ 6.87\ M\Omega/m$. This impedance is independent of the frequency, and appears to be negative inductive, because of $a < b$. The complication relevant to this impedance will be discussed in other report.

3.3 Broad band resonator

In general a resonator type impedance can be written as,

$$Z_T(\omega) = \frac{R_T}{1 + jQ(\omega/\omega_R - \omega_R/\omega)} \tag{3-4}$$

where R_T is the transverse shunt resistance, Q is the quality factor and ω_R is the resonant frequency. The most important broad band resonator type impedance is the one caused by the vacuum chamber discontinuities, such as the bellows. This type of impedance has been extensively studied, and the consensus is to model it as a resonator with $\omega_R \approx c/b$, and $Q = 1$, which ranged as a longitudinal broad band impedance of $Z_L(n)/n$ from j 5 Ω to j 30 Ω. See [3] for CERN PS and Booster, [4] for ISR, [5] for AGS, and even SPS [6]. This impedance is inductive.

This type of impedance at the NSNS is shown in Fig.3a for longitudinal, where at the low frequency we have $Z_L(n)/n = j$ 25 Ω. The transverse counter part can be found by using

$$Z_T(n) = \frac{2c}{b^2\omega_0} Z_L(n)/n \tag{3-5}$$

which is shown in Fig.3b, where at low frequency, we have $Z_T(n) \approx j$ 200 $K\Omega/m$. The real part of this impedance is not very large at the low frequency range, which is also shown.

3.4 Narrow band resonator

This type of impedance includes mainly the RF cavities. Usually the longitudinal impedance of the cavities is known. It is claimed [7] that the Panofsky-Wenzel theorem can be applied for narrow band impedance,

$$Z_T(n) = \frac{c}{\omega_0} Z_L(n)/n \tag{3-6}$$

which will be used in this report, disregarding the fact that the result obtained by this formulation is not very satisfactory.

For the high order mode of the RF cavities, the measurement is indispensable.

3.5 Low frequency impedance

The low frequency impedance includes mainly the extraction kicker, which can be seen as a resonator, or a group of resonators, at a resonant frequency higher than the fundamental RF frequency, but much lower than the broad band impedance. The quality factor is also much smaller than the RF cavities. One of such kind of model can be from [8], with the resonant frequency, the transverse shunt resistance, and the quality factor picked up at, say, $\omega_R = 2\pi \times 15\ MHz$, $R_T = 70\ K\Omega/m$, and $Q = 0.7$. Note that the corresponding longitudinal impedance at $n = 1$ is only about $Z_L(n)/n = 1.3\ \Omega$.

This impedance is shown in Fig.4a and Fig.4b for longitudinal and transverse, respectively.

4 Chromatic Effect

The machine chromaticity ξ is defined in the following relation,

$$\xi = \frac{\Delta\nu}{\nu} / \frac{\Delta p}{p} \tag{4-1}$$

where $\Delta\nu$ and Δp are betatron tune and momentum deviations, respectively. If we define the chromatic frequency

$$\omega_\xi = \frac{\xi}{\eta}\nu\omega_0 \tag{4-2}$$

then the beam spectrum line n' should be using,

$$n' = n - \frac{\omega_\xi}{\omega_0} \tag{4-3}$$

The machine chromaticity, therefore, is effective in the instability study if we simply substitute (4-3) into (2-3). Graphically, the beam spectrum just shifts by the chromatic frequency ω_ξ.

For the NSNS [2], the vertical chromaticity $\xi_v \nu_v = -7.3$. The full beam momentum spread can be as large as 1.7% [9], chosen by the consideration of the longitudinal microwave instability. The full incoherent tune spread due to the chromaticity is 0.124, which is large enough to call for chromaticity correction. Adding sextupoles will pose dynamic aperture limitation, however, it also provides flexibility in the machine operation.

5 Beam Instability

We note that the space charge impedance is purely imaginary, therefore, it generates frequency shift. In the following, only the instabilities caused by the real part of the wall resistivity, the broad band, the narrow band, and the low frequency impedances will be evaluated. As by-product, the frequency shift due to the imaginary part of these impedances will also be shown.

5.1 Resistive wall instability

The wall resistance is an important one among these impedances. At the betatron tune of $\nu_v = 5.82$, the real part of the effective negative wall resistance is about $-12.31\ K\Omega/m$, which gives rise to an instability with the growth time of less than 1 ms, at the worst case. In Fig.5a, this is shown by varying the machine chromaticity from $\xi = -0.5$ to 0.5. At the negative chromaticity, this instability should be eliminated. The mode $m = 1$ resistive wall instability is also shown, where a little complication around the zero chromaticity can be observed. For some machine this is of importance, but it is not so for the NSNS, since the worst growth time is only about 5 ms.

The resistive wall impedance induced frequency shift is shown in Fig.5b, which is negligible.

4 Chromatic Effect

The machine chromaticity ξ is defined in the following relation,

$$\xi = \frac{\Delta\nu}{\nu} / \frac{\Delta p}{p} \tag{4-1}$$

where $\Delta\nu$ and Δp are betatron tune and momentum deviations, respectively. If we define the chromatic frequency

$$\omega_\xi = \frac{\xi}{\eta}\nu\omega_0 \tag{4-2}$$

then the beam spectrum line n' should be using,

$$n' = n - \frac{\omega_\xi}{\omega_0} \tag{4-3}$$

The machine chromaticity, therefore, is effective in the instability study if we simply substitute (4-3) into (2-3). Graphically, the beam spectrum just shifts by the chromatic frequency ω_ξ.

For the NSNS [2], the vertical chromaticity $\xi_v\nu_v = -7.3$. The full beam momentum spread can be as large as 1.7% [9], chosen by the consideration of the longitudinal microwave instability. The full incoherent tune spread due to the chromaticity is 0.124, which is large enough to call for chromaticity correction. Adding sextupoles will pose dynamic aperture limitation, however, it also provides flexibility in the machine operation.

5 Beam Instability

We note that the space charge impedance is purely imaginary, therefore, it generates frequency shift. In the following, only the instabilities caused by the real part of the wall resistivity, the broad band, the narrow band, and the low frequency impedances will be evaluated. As by-product, the frequency shift due to the imaginary part of these impedances will also be shown.

5.1 Resistive wall instability

The wall resistance is an important one among these impedances. At the betatron tune of $\nu_v = 5.82$, the real part of the effective negative wall resistance is about $-12.31\ K\Omega/m$, which gives rise to an instability with the growth time of less than 1 ms, at the worst case. In Fig.5a, this is shown by varying the machine chromaticity from $\xi = -0.5$ to 0.5. At the negative chromaticity, this instability should be eliminated. The mode $m = 1$ resistive wall instability is also shown, where a little complication around the zero chromaticity can be observed. For some machine this is of importance, but it is not so for the NSNS, since the worst growth time is only about 5 ms.

The resistive wall impedance induced frequency shift is shown in Fig.5b, which is negligible.

4. The low frequency impedances, such as the kickers, possibly gives rise to strong instability, which, once again, can be eliminated by the chromaticity adjustment. Also, the measurement is needed in a further study of this instability.
5. Finally, the necessity of the chromaticity correction may need more attention than the transverse instabilities.

References

[1] S.Y. Zhang, AGS Tech. Notes, No.422, BNL, Nov. 1995.

[2] Y.Y. Lee, NSNS Tech. Notes, No.26, BNL, Feb. 1997.

[3] J. Gareyte and F. Sacherer, Proc. 9th Int. Conf. on High Energy Accelerators, p.341, 1974.

[4] P. Bramham et. al. IEEE Trans. NS, Vol.NS-24,No.3, p.1490, 1977.

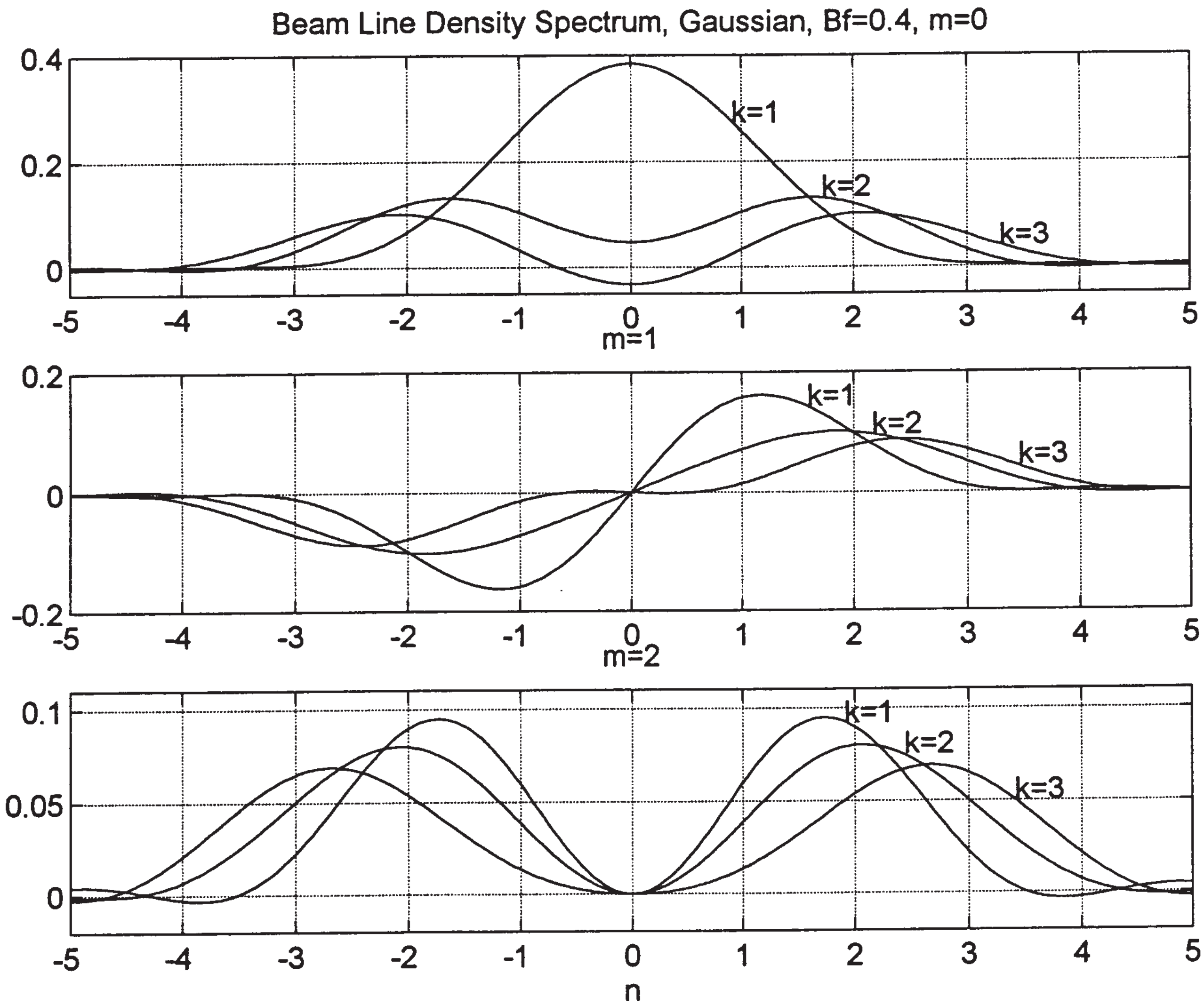
[5] F. Pedersen and E. Raka, IEEE Trans. NS, Vol.NS-26,No.3, p.3592, 1979.

[6] D. Boussard and J. Gareyte, SPS/AC/DB/JG/EEK, Improvement Report No.81, CERN 1980.

[7] A. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, Wiley, New York, 1994.

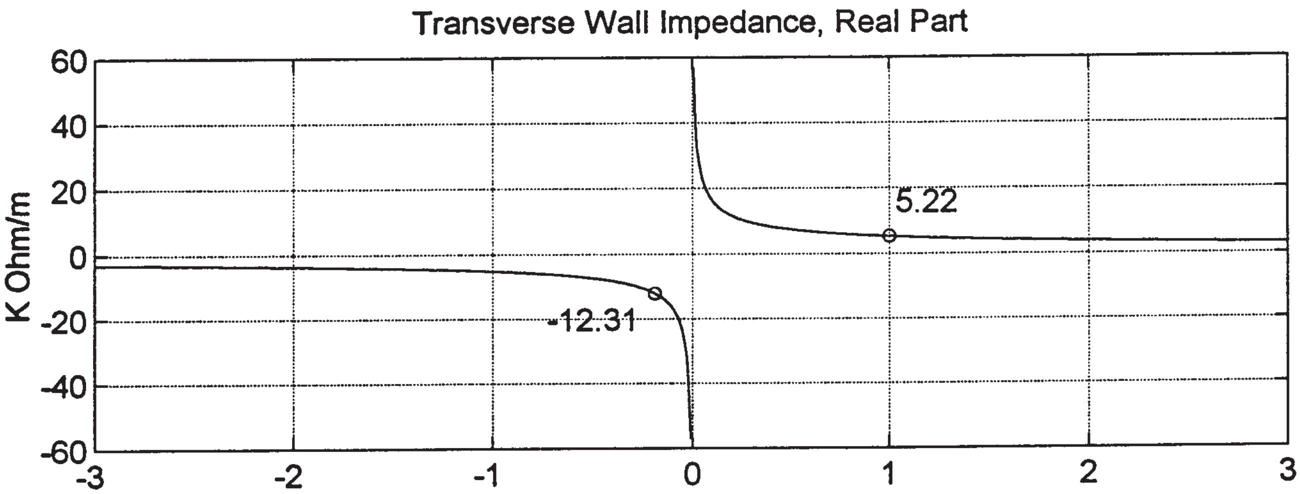
[8] G. Nassibian and F. Sacherer, Nucl. Inst. Meth. Vol.159, p21, 1979.

[9] A.G. Ruggiero, NSNS Tech. Notes, No.7, BNL, June 1996.



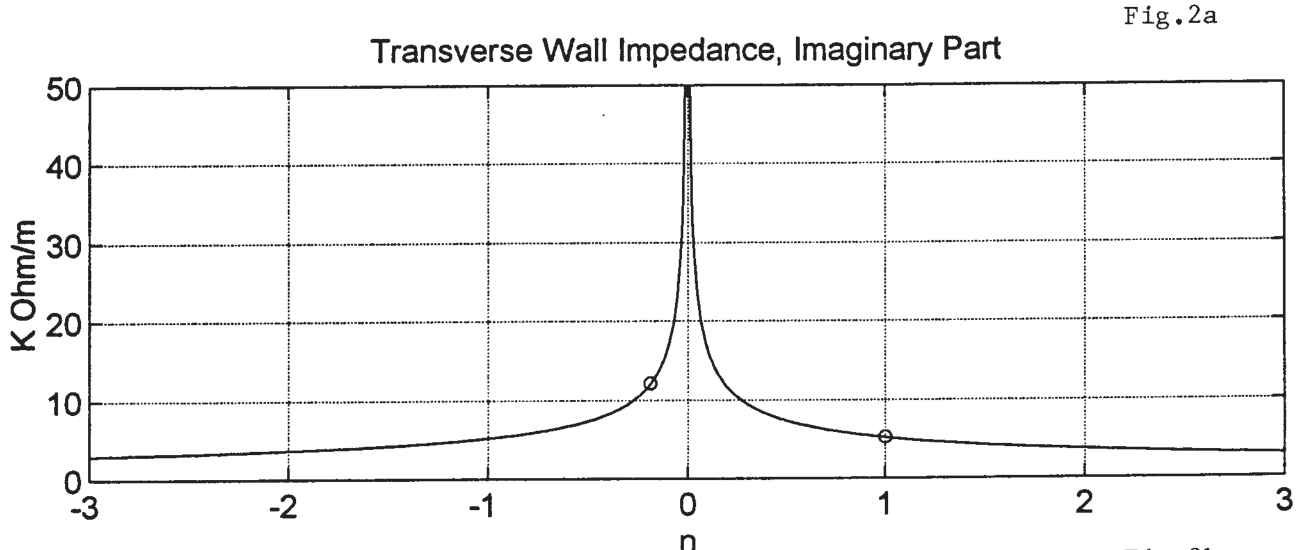
Beam Line Density Spectrum, Gaussian, Bf=0.4, m=0
0.4
0.2
0
k=1
k=2
k=3
-5
-4
-3
-2
-1
0
1
2
3
4
5
m=1
0.2
0
-0.2
k=1
k=2
k=3
m=2
0.1
0.05
0
k=1
k=2
k=3
n

Fig.1



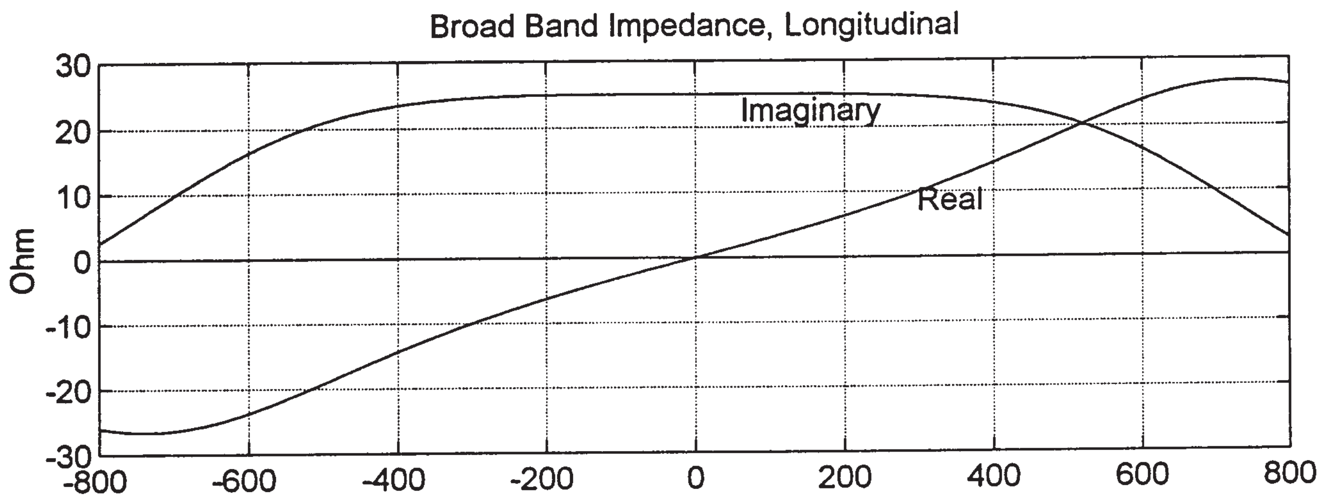
Transverse Wall Impedance, Real Part
K Ohm/m
60
40
20
0
-20
-40
-60
5.22
-12.31
-3
-2
-1
0
1
2
3

Fig.2a



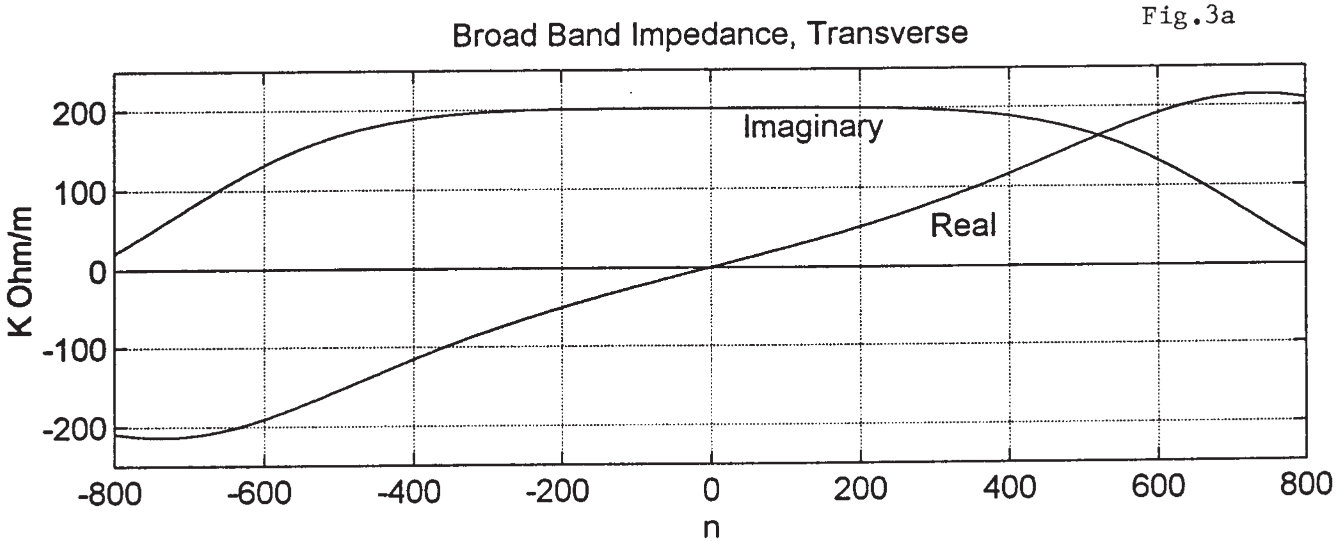
Transverse Wall Impedance, Imaginary Part
K Ohm/m
50
40
30
20
10
0
-3
-2
-1
0
1
2
3
n

Fig.2b



Broad Band Impedance, Longitudinal
Ohm
30
20
10
0
-10
-20
-30
-800
-600
-400
-200
0
200
400
600
800
Imaginary
Real

Fig.3a



Broad Band Impedance, Transverse
K Ohm/m
200
100
0
-100
-200
-800
-600
-400
-200
0
200
400
600
800
n
Imaginary
Real

Fig.3b

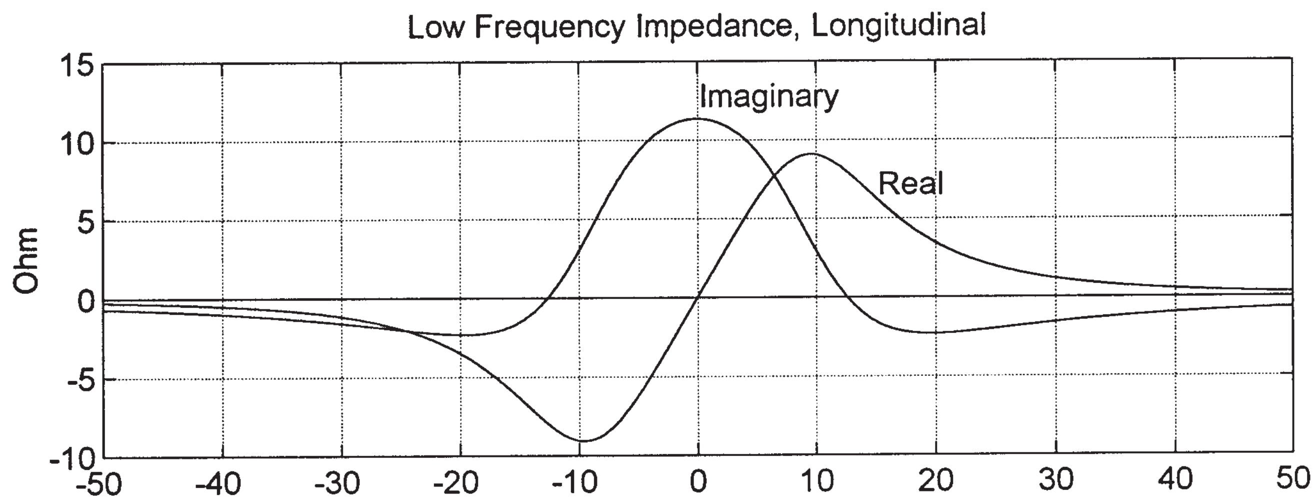

Low Frequency Impedance, Longitudinal
Imaginary
Real
Ohm
15
10
5
0
-5
-10
-50
-40
-30
-20
-10
0
10
20
30
40
50

Fig.4a

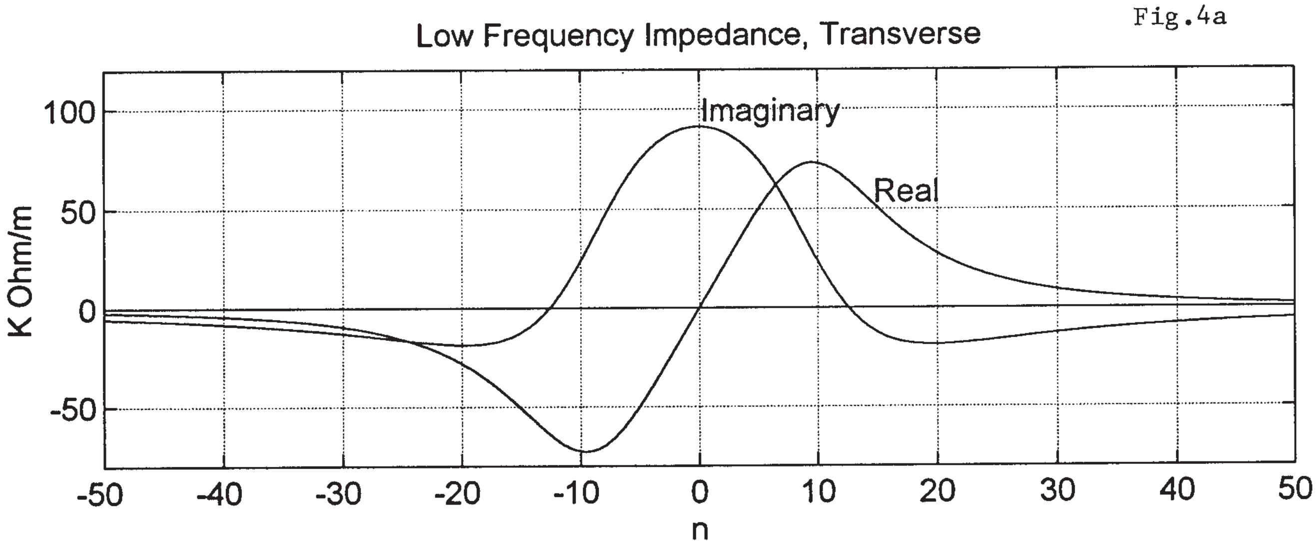

Low Frequency Impedance, Transverse
Imaginary
Real
K Ohm/m
100
50
0
-50
-50
-40
-30
-20
-10
0
10
20
30
40
50
n

Fig.4b

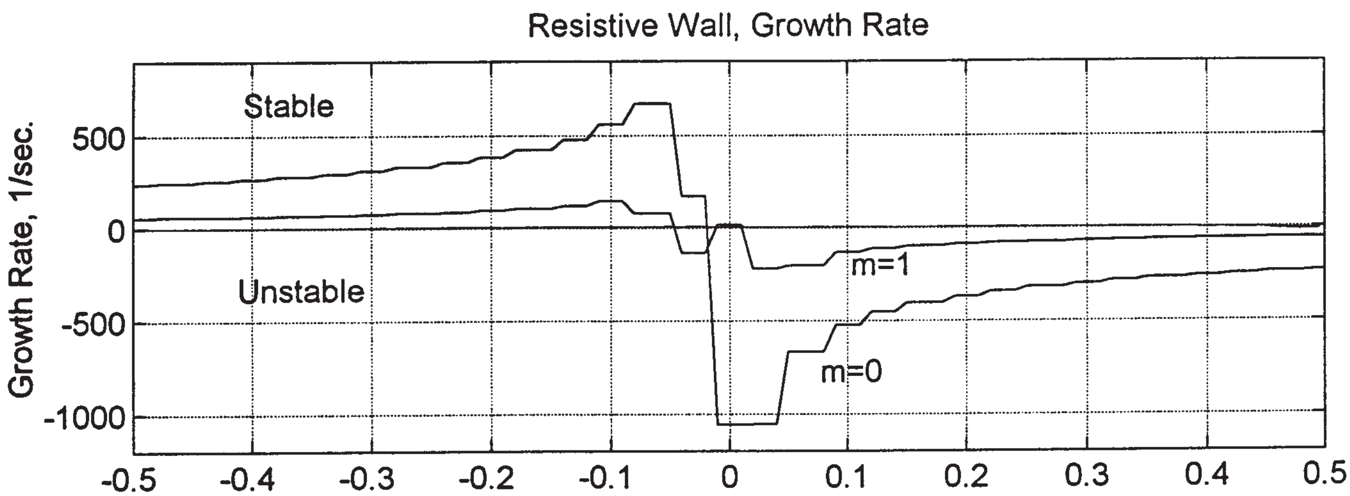

Resistive Wall, Growth Rate
Growth Rate, 1/sec.
Stable
Unstable
m=1
m=0
500
0
-500
-1000
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

Fig.5a

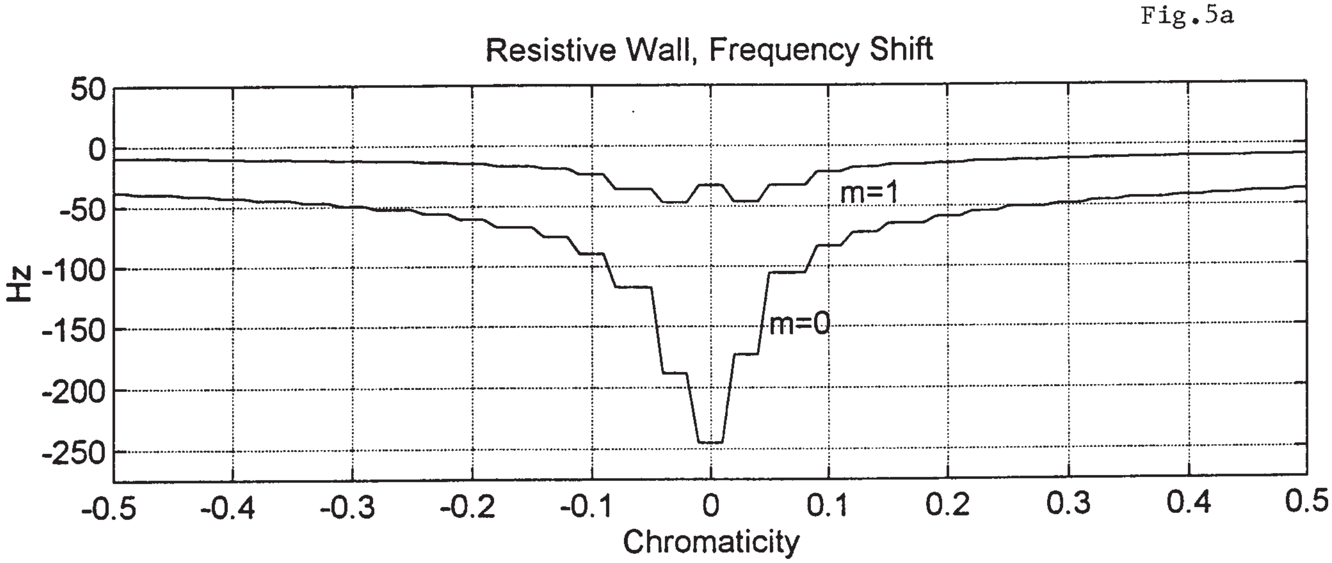

Resistive Wall, Frequency Shift
Hz
m=1
m=0
50
0
-50
-100
-150
-200
-250
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
Chromaticity

Fig.5b

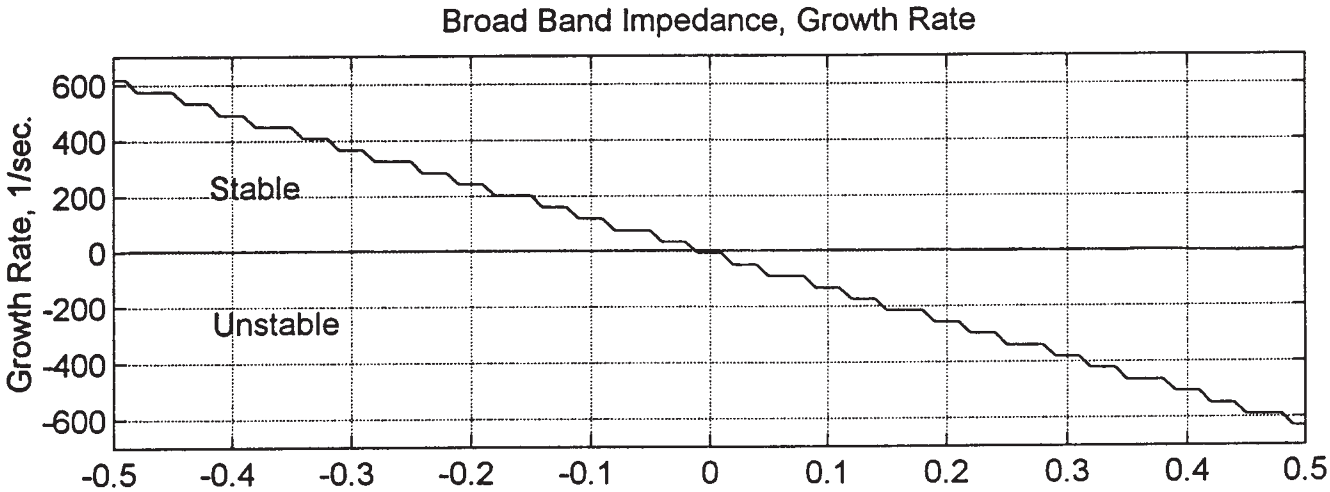

Broad Band Impedance, Growth Rate
Growth Rate, 1/sec.
600
400
200
0
-200
-400
-600
Stable
Unstable
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

Fig.6a

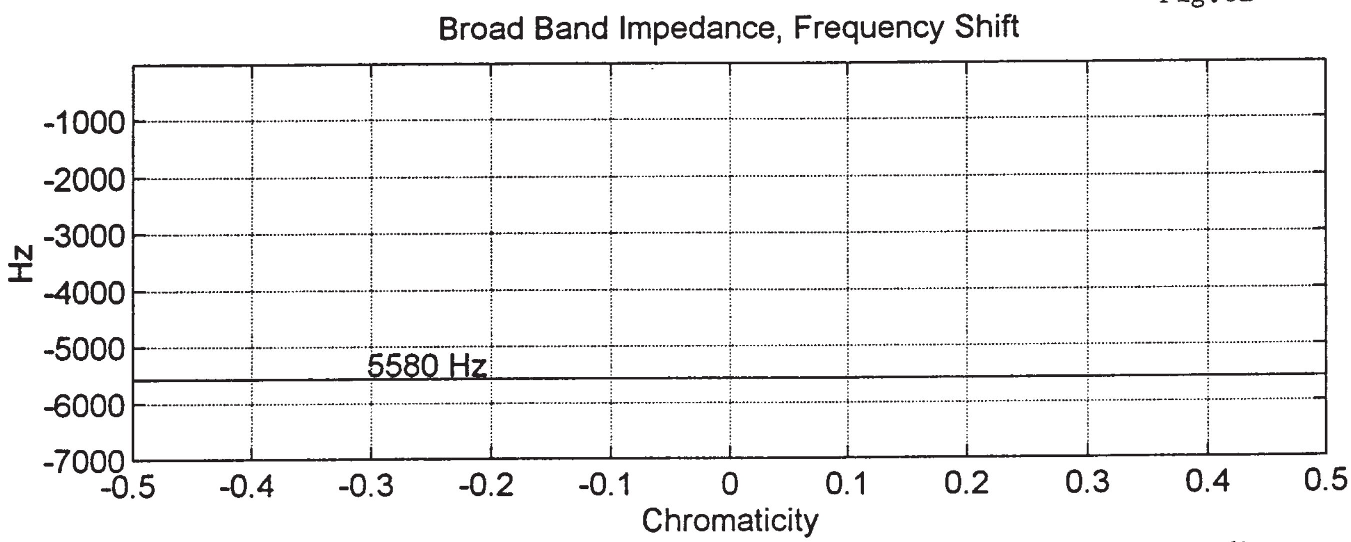
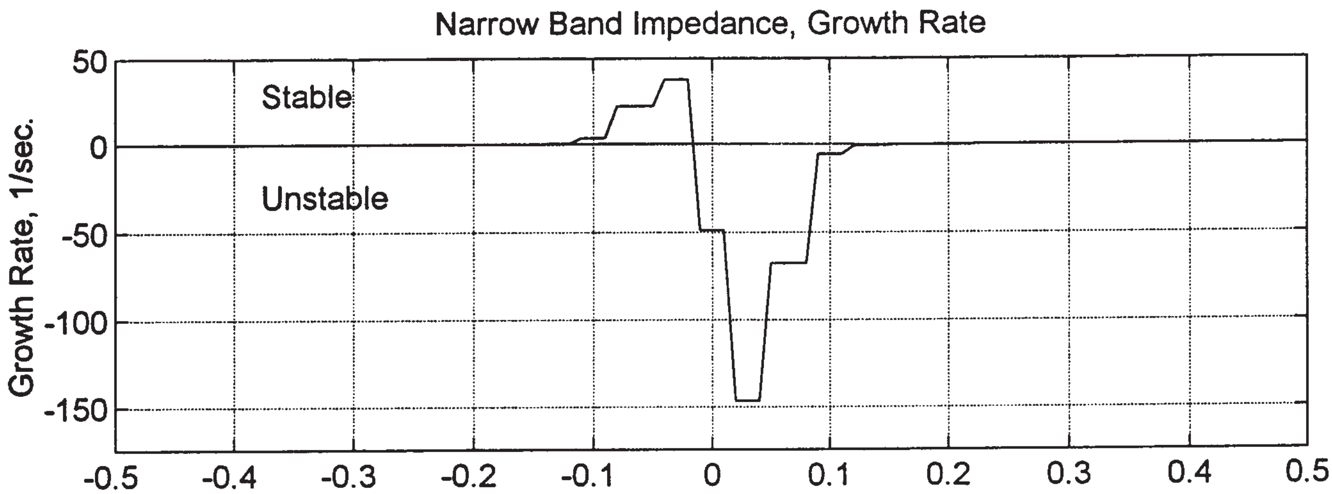
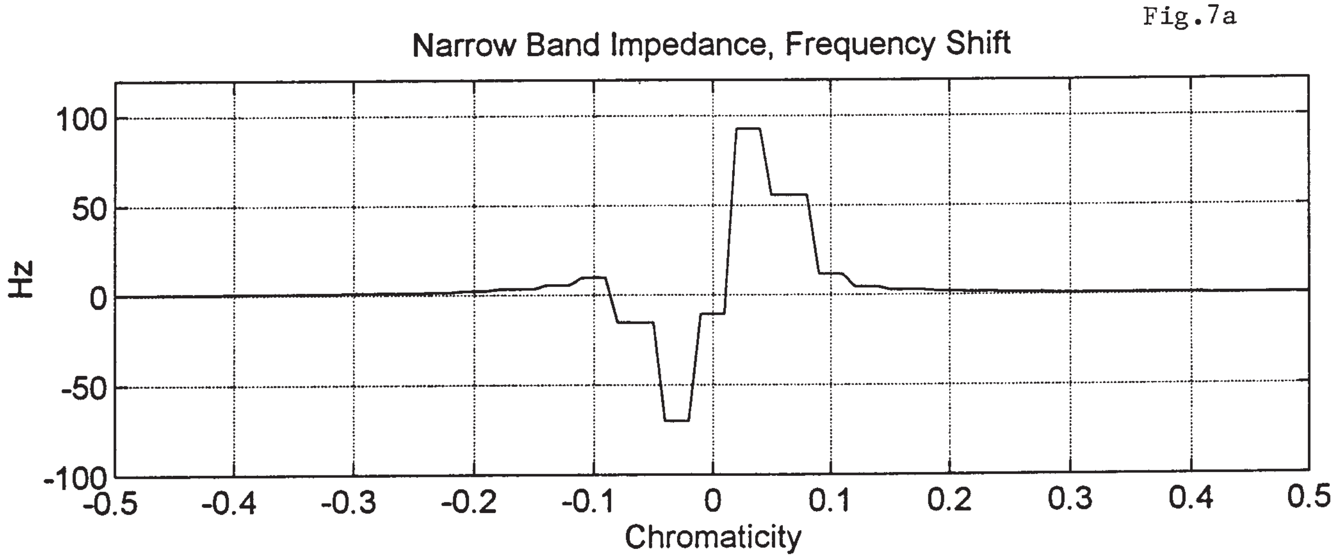

Broad Band Impedance, Frequency Shift
Hz
-1000
-2000
-3000
-4000
-5000
-6000
-7000
5580 Hz
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
Chromaticity

Fig.6b



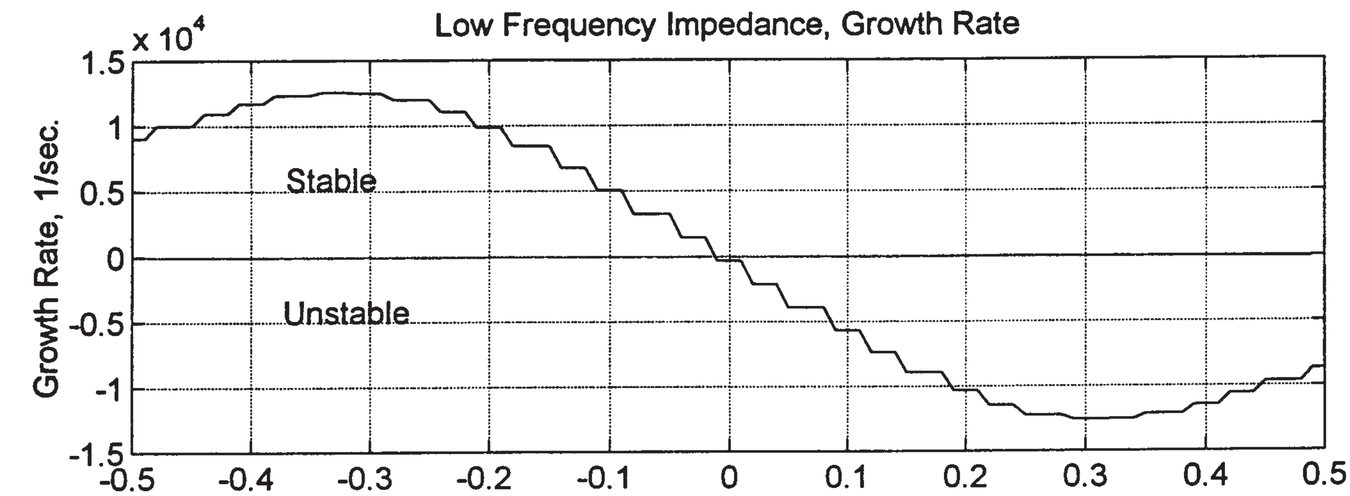
Narrow Band Impedance, Growth Rate
Growth Rate, 1/sec.
50
0
-50
-100
-150
Stable
Unstable
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

Fig.7a



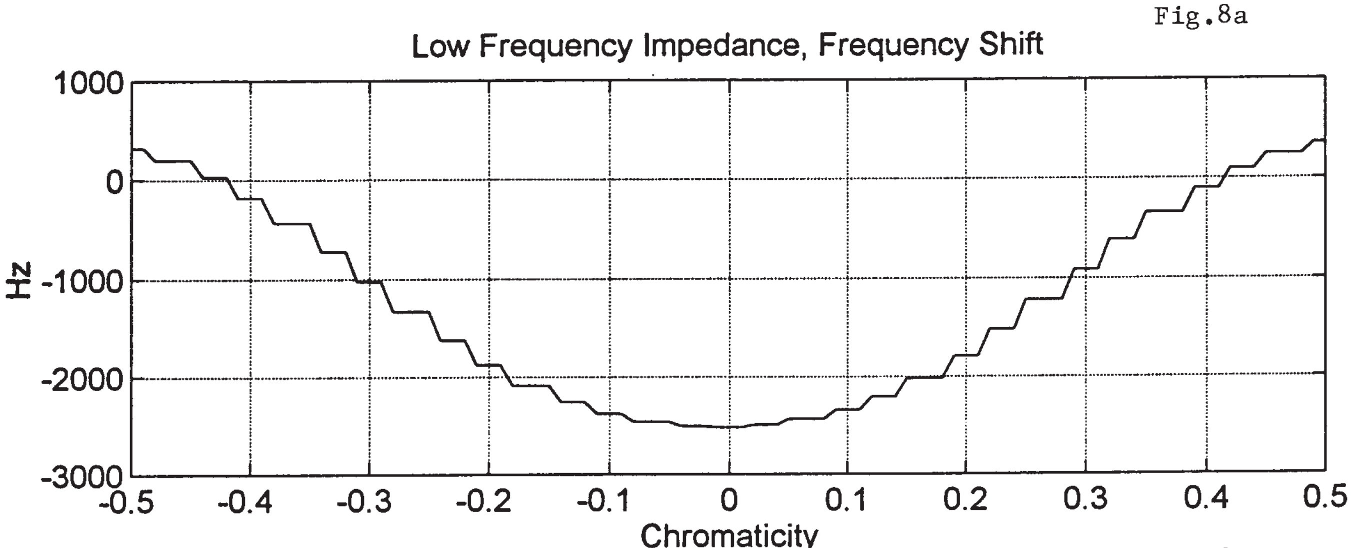
Narrow Band Impedance, Frequency Shift
Hz
100
50
0
-50
-100
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
Chromaticity

Fig.7b



Low Frequency Impedance, Growth Rate
x 10^4
Growth Rate, 1/sec.
Stable
Unstable
1.5
1
0.5
0
-0.5
-1
-1.5
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

Fig.8a



Low Frequency Impedance, Frequency Shift
Hz
1000
0
-1000
-2000
-3000
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
Chromaticity

Fig.8b