NSNS TRANSVERSE MICROWAVE INSTABILITIES

BNL/NSNS TECHNICAL NOTE

NO. 034

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June 6, 1997

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1 Introduction

In this note, the conventional space charge coherent and incoherent tune shifts will be presented. It will be shown that applying the transverse space charge impedance to the beam dynamic equation, we get the difference of the space charge coherent and incoherent tune shifts.

Secondly, the transverse Landau damping is discussed with respect to the sources of the coherent tune shift and the incoherent tune shift. It is indicated that for low energy synchrotrons, the space charge incoherent tune spread is an important stabilizing force. Also it is shown that the transverse space charge impedance is relevant to both the coherent tune shift and the stabilizing incoherent tune spread.

Finally, the NSNS transverse microwave instability is studied. Because of the large space charge tune spread, the transverse microwave instability is not of serious concern. For the same reason, other damping factors, i.e. the chromatic, the frequency slippage, the octupolar, and the synchrotron oscillation effects are not critical in the transverse microwave instability. On the other hand, the incoherent tune spread cannot help for the rigid bunch instabilities, because of the synchrotron oscillation. Careful study for this kind of instabilities, such as the resistive wall instability, is needed.

2 Coherent and Incoherent Tune Shifts

For coasting symmetric beam with non-penetrating fields, the space charge incoherent and coherent tune shifts are defined as [1],

$$\Delta\nu_{inc} = \frac{-NRr_0}{\pi\nu_0\beta^2\gamma}\left(\frac{\epsilon_1}{b^2} + \beta^2\frac{\epsilon_2}{g^2} + \frac{1}{2a^2\gamma^2}\right) \tag{1}$$

and

$$\Delta\nu_{coh} = \frac{-NRr_0}{\pi\nu_0\beta^2\gamma}\left(\beta^2\frac{\epsilon_1}{b^2} + \beta^2\frac{\epsilon_2}{g^2} + \frac{\xi_1}{b^2\gamma^2}\right) \tag{2}$$

where N is the total number of particles, R is the machine average radius, r_0 is the classical radius of proton, ν_0 is the betatron tune with zero beam current, and b is the average half chamber height, g is the half pole gap, a is the average radius of the beam, and ϵ_1 and ϵ_2

are the Laslett incoherent electric and magnetic coefficients, respectively. The coefficient ξ_1 is the Laslett coherent electric coefficient.

For the simplified model, we consider circular chamber, which gives rise to,

$$\epsilon_1 = \epsilon_2 = 0 \tag{3}$$

and

$$\xi_1 = 0.5 \tag{4}$$

then the incoherent and coherent tune shifts become,

$$\Delta\nu_{inc} = \frac{-NRr_0}{2\pi\nu_0\beta^2\gamma^3a^2} \tag{5}$$

and

$$\Delta\nu_{coh} = \frac{-NRr_0}{2\pi\nu_0\beta^2\gamma^3b^2} \tag{6}$$

We indicate that for low energy synchrotrons, since γ is small, therefore, the equations (5) and (6) are approximately right, even the chamber is not circular.

For bunched beams, we take a simplified approach, by adding the bunching factor B_f to the denominators of equations (5) and (6).

3 Transverse Space Charge Impedance

The conventional transverse space charge impedance is defined as [2,3],

$$Z_{TSC} = j\frac{RZ_0}{\beta^2\gamma^2}(\frac{1}{b^2} - \frac{1}{a^2}) \tag{7}$$

where Z_0 is the impedance of free space. Since $a < b$, this impedance is usually taken as capacitive.

Consider the transverse bunched beam dynamic equation with the azimuthal mode $m = 0$ [4],

$$\omega - \omega_\beta = \frac{j\beta eI_0}{2Rm_0\gamma\nu_0\omega_0}\sum_{n=-\infty}^{\infty} Z_T(n)\Lambda_0^2(n) \tag{8}$$

where ω_β and ω_0 are the betatron and revolution frequencies, respectively, I_0 is the average beam current, and m_0 is the rest mass of proton. Also Z_T is the transverse impedance, and Λ_0 is the spectrum of the beam line density for $m = 0$ mode.

In the following, we show that,

- The tune shifts shown in (5) and (6) can be obtained by substituting a proper part of the space charge impedance into the dynamic equation (8).

- The transverse space charge impedance represents the difference between the coherent and incoherent tune shifts.
- The incoherent tune shift will be cancelled in the dynamic equation, and therefore, it pays no role in the coherent motion.

3.1 Impedance and dynamic equation

First, we take the chamber part of the transverse space charge impedance (7),

$$Z_T = j\frac{RZ_0}{\beta^2\gamma^2}\frac{1}{b^2} \tag{9}$$

For coasting beams, the beam line density is a DC signal, therefore, its spectrum is a delta function. The amplitude of this delta function can be found in [4] as $1/2\pi$, i.e.

$$\Lambda_0^2(n) = \frac{1}{2\pi}\delta(n) \tag{10}$$

Thus, the summation in (8) is removed.

Now we use

$$I_0 = \frac{Ne\omega_0}{2\pi} \tag{11}$$

and

$$Z_0 = \frac{1}{\epsilon_0 c} \tag{12}$$

where ϵ_0 is the permittivity in free space. Also using

$$r_0 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_0c^2} \tag{13}$$

and

$$\omega_0 = \frac{\beta c}{R} \tag{14}$$

then the tune shift by substituting (9) into (8) is

$$\Delta\nu = \frac{-NRr_0}{2\pi\nu_0\beta^2\gamma^3b^2} \tag{15}$$

which is exactly the space charge coherent tune shift shown in (6). This shows that the chamber part of the space charge impedance represents the coherent tune shift. Similarly, the beam part of the impedance is relevant to the incoherent tune shift.

3.2 Transverse space charge impedance

Now substituting the transverse space charge impedance (7) into the dynamic equation (8), we have,

$$\omega - \omega_\beta = \Delta\nu_{coh}\omega_0 - \Delta\nu_{inc}\omega_0 \tag{16}$$

i.e. the transverse space charge impedance represents the difference between the coherent and incoherent tune shifts.

The impedance (7) is defined based on the deflecting electromagnetic fields distributed between the beam and the perfectly conducting wall [5]. The physical implication of this impedance to the beam, however, is only shown in (16).

Specifically, substituting the transverse space charge impedance into the dynamic equation gives neither coherent nor incoherent tune shift. In the case that $a \ll b$, the obtained tune shift will be approximately equal to the incoherent tune shift. However, this tune shift is increased, while the space charge incoherent tune shift should be decreased.

3.3 Incoherent tune shift and coherent motion

Using the transverse impedance (7), it can be shown that the incoherent tune shift plays no role for the coherent motion in the dynamic equation (8).

Writing on the left side of (8) by the following convention ,

$$\omega_\beta = \omega_{\beta 0} + \Delta\nu_{inc}\omega_0 \tag{17}$$

and also using (16), the equation (8) becomes,

$$\omega - \omega_{\beta 0} - \Delta\nu_{inc}\omega_0 = \Delta\nu_{coh}\omega_0 - \Delta\nu_{inc}\omega_0 \tag{18}$$

where the incoherent tune shift is cancelled. This shows that the incoherent tune shift plays no role in the transverse coherent motion. Therefore, the beam part of the transverse space charge impedance is a redundancy.

The writing of (17) is following the longitudinal case, where the synchrotron oscillation frequency has to be written as $\omega_S = \omega_{S0} + \Delta\nu_{S,inc}\omega_0$, because the incoherent frequency shift affects the longitudinal focusing, which is often called the potential well effect. In the transverse case, the similar effect is negligible.

4 Transverse Landau Damping

For NSNS storage ring, the bunch is long. Also, since the beam stays in the ring for about 1 ms, only the transverse microwave instabilities having growth rate greater than the synchrotron frequency is relevant. Therefore, the coasting beam criterion can be applied. The power spectrum of the perturbation will be a delta function $\delta(n - n_1)/2\pi$, where the spectrum line n_1 represents the frequency $(n_1 + \nu_0)\omega_0$, because only the perturbation at these frequencies has a chance to grow. Substituting the beam peak current I_p for the average current I_0, the equation (8) becomes,

$$\omega - \omega_\beta = \frac{j\beta e I_p}{4\pi R m_0 \gamma \nu_0 \omega_0} Z_T(n_1) \tag{19}$$

To proceed further, we write the left side of the equation (19) as the frequency spread $\Delta\omega$, which will be responsible for the Landau damping, and can be written by the tune spread $\Delta\nu$ as,

$$\Delta\omega = \Delta\nu\omega_0 \tag{20}$$

Note that the Landau damping has two implications,

- If the impedance is real and positive, the system is stable, and the Landau damping is not needed. If it is negative, then the frequency spread must be larger than the growth rate to suppress the instability.
- If the impedance is imaginary, then the frequency spread on the left side must be larger than the coherent frequency shift. Otherwise, an infinitesimal perturbation may cause instability.

The microwave instability criterion is, therefore, obtained as follows,

$$\Delta\nu > \frac{\beta e I_p}{4\pi R m_0 \gamma \omega_\beta \omega_0} |Z_T(n_1)| \tag{21}$$

It remains to clarify the sources responsible for the incoherent and coherent tune shifts.

4.1 Incoherent tune spread

For the incoherent tune spread, we consider the following sources,

- Space charge incoherent tune spread, which is the largest stabilizing force for the low energy proton synchrotrons. For the high energy machine, the tune spread is decreased, and its contribution diminishes. This is a reason that the transverse instabilities is more critical for the high energy machines.
- Chromatic tune spread. A large chromatic tune spread will cause beam loss due to the resonance crossing, therefore, it needs to be corrected. For bunched beams, the chromatic tune spread is not effective for the weak instabilities with the growth rate comparable to the synchrotron oscillation period. It is, however, effective for the fast instabilities. This tune spread is momentum spread dependent.
- Frequency slippage. This is momentum spread dependent. This tune spread could be cancelled by the chromatic tune spread, then the trick is to let the cancellation happen at a stable frequency region.
- Octupolar tune spread. This tune spread is betatron oscillation amplitude dependent. In case the microwave instability is a problem, octupoles can be added to ensure the beam stability.

- Finally, for bunched beams, the synchrotron oscillation tune spread may help. Conventionally, this contribution is estimated as $\Delta\omega \approx \omega_S = \Delta\nu_S\omega_0$.

The combined tune spread can, therefore, be written for the effective frequency $(n_1+\nu_0)\omega_0$ as [6],

$$\Delta\omega = ((n_1 + \nu_0)\eta - \xi\nu_0)\frac{\Delta p}{p} + \Delta\nu_{inc} + \Delta\nu_{oct} + \Delta\nu_S)\omega_0 \qquad (22)$$

where $\Delta p/p$ is the beam momentum spread. Usually, the slippage η and the chromaticity ξ should have same sign, such that the cancellation of their contributions will happen at the stable region. This could be important for high energy machines, but not for the NSNS.

4.2 Coherent tune shift

The coherent tune shifts simply come from mainly two sources,

- Space charge coherent tune shift.
- Broad band impedance induced tune shift.

It can be observed that if the conventional transverse space charge impedance (7) is used, then it is relevant to both the incoherent and coherent tune shifts. Therefore, it takes effect on both sides of the equation (21).

However, the transverse microwave instability can be estimated by taking the sum of the transverse space charge impedance with the broad band impedances, such as in [8]. If the sum is negative, then the incoherent tune spread is dominant, and the system is stable. If it is positive, then the space charge incoherent tune spread is not strong enough to stabilize the system by itself.

Note that this approach is not only valid for the low energy synchrotron, but also can be used for the high energy rings. For high energy synchrotrons, the image effect is often stronger than the direct effect. However, the image coherent tune shift is approximately cancelled by the image incoherent tune spread, therefore, the total effect is still negligible.

5 NSNS Transverse Microwave Instabilities

The criterion of the transverse microwave instability in (21) will be used to analyze the NSNS transverse instabilities.

As shown in [7], the real part of the broad band impedance is much smaller than the imaginary part in the relevant frequency range, therefore, we consider only the coherent tune shift. In this way, the broad band impedance will be considered as $Z_{TBB}(n) \approx j\ 200\ K\Omega/m$, as in [7]. If we use (9) to get the space charge coherent impedance, then for $b = 10\ cm$, we have the coherent part of the space charge impedance $Z_{TSCcoh}(n) \approx j\ 406\ K\Omega/m$. For 2 MW storage ring, the peak current is $I_p = 91\ A$. We also have $R = 35.124\ m$, $\nu_0 = 5.82$, and $\omega_0 = 2\pi \times 1.189 \times 10^6\ rad./\sec$. Using (21), we get the coherent tune shift as

$$\Delta\nu_{coh,total} = -0.016 \tag{23}$$

where the contribution of the broad band impedance is -0.005, and the space charge coherent tune is -0.011.

The space charge incoherent tune spread can be calculated either using (5) or substituting the space charge incoherent impedance into the equation (19). If we use the latter, then taking $a = \sqrt{2}\sigma = 2.36\ cm$ [7], we get the incoherent part of the space charge impedance $Z_{TSCinc}(n) \approx j\ 7.3\ M\Omega/m$. Thus, the space charge incoherent tune spread is

$$\Delta\nu_{inc} = -0.20 \tag{24}$$

It is clear that $|\Delta\nu_{inc}| \gg |\Delta\nu_{coh,total}|$, therefore, for NSNS the transverse microwave instability will not be of serious concern. Also since the space charge incoherent tune spread is relatively large, other sources such as the frequency slippage and chromatic effects are not critical for the beam stabilization.

This mechanism of stabilization of the transverse microwave instability is, in fact, applicable to all low energy synchrotrons. In specific, no special effort is needed for the Landau damping of the transverse microwave instability for a low energy synchrotron.

For rigid bunch instabilities, such as the resistive wall instability or the instability caused by the kicker impedance, the incoherent tune spread is not effective in terms of the Landau damping, because of the synchrotron oscillation. Also, the machine has to be operated at a small negative chromaticity, because of the large beam momentum spread required by the depression of the longitudinal microwave instability. Therefore, this instability needs attentions.

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