



Estimates of the linear coupling effects

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Linear coupling effects

In this note the effects of linear coupling are estimated for the proposed operating point, $\nu_x = 6.23$, $\nu_y = 6.20$ suggested by A. Fedotov [1, 2].

Two linear coupling effects are measurable and are used to correct linear coupling due to skew quadrupole field errors. In this note, the magnitude of these two effects will be computed for the proposed operating point $\nu_x = 6.23$, $\nu_y = 6.20$.

In the absence of skew quadrupole field errors, the linear horizontal motion has the tune ν_x , and the linear vertical motion has the tune ν_y . In the presence of the skew quadrupole field errors the horizontal and vertical motions each show the presence of two tunes, ν_1 and ν_2 . Theory indicates that there are now two modes present, one with the tune ν_1 and the other with the tune ν_2 . The difference $|\nu_1 - \nu_2|$ is called the tune splitting. ν_1 and ν_2 and the tune splitting can be measured. Global correction of the linear coupling effects is done by adjusting the skew quadrupole field correctors to minimize the tune splitting. It is estimated below that the increase in tune splitting caused by a rms rotation error of 1 mr in the location of the quadrupoles is about .0012 for the proposed tune of 6.23, 6.20. This is a small increase in the tune splitting compared to the tune splitting of .030 present without the coupling.

A second linear coupling effect has to do with the coupling of the horizontal and vertical motions. One finds that in one of the modes, the horizontal motion is larger than the vertical motion, while in the other mode the vertical motion is larger. Let us assume that the mode with the larger horizontal motion is the mode with the tune ν_1 , and the mode with the larger vertical motion is the mode with the tune ν_2 . The local coupling at a particular

location in the ring is described by the two parameters C_1 and C_2 which are defined as follows: If one excites the betatron motion so that only the ν_1 mode is present then

$$C_1 = [y_{max}/x_{max}]_1 \quad (1)$$

where x_{max} and y_{max} are defined as the maximum values of the horizontal and vertical motions. Similarly, C_2 is defined as

$$C_2 = [x_{max}/y_{max}]_2 \quad (2)$$

C_1 and C_2 are measurable at any point in the ring where there are beam position monitors. They can be used to locally correct the coupling at that point by adjusting the skew quadrupole field correctors to minimize C_1 and C_2 [3, 4, 5]. It is estimated below that the local coupling caused by a rms rotation error of 1 mr in the location of the quadrupoles is about 14% for the proposed tune of 6.23, 6.20. This is probably small compared to the non linear coupling due to non linear space charge forces and the chromaticity correcting sextupoles.

Tune splitting estimate

ν_1 and ν_2 can be computed from [6]

$$\begin{aligned} \nu_1 - \nu_x &= -(\nu_x - \nu_y)/2 + [((\nu_x - \nu_y)/2)^2 + |\Delta\nu|^2]^{1/2} \\ \nu_2 - \nu_y &= -(\nu_1 - \nu_x) \\ \Delta\nu &= \frac{1}{4\pi} \int ds (\beta_x \beta_y)^{1/2} \frac{B_1}{B\rho} \exp[i(\psi_x - \psi_y)] \\ B_x &= B_1 x \end{aligned} \quad (3)$$

which holds when $\nu_x \geq \nu_y$ and ν_x, ν_y are near the $\nu_x = \nu_y$ resonance line. $B_x = B_1 x$ is the skew quadrupole field on the median plane. In the quadrupoles, $B_1 = 2G\Delta\theta$ where $\Delta\theta$ is the rotation error in the quadrupoles and G the quadrupole field gradient. The tune splitting depends on $\Delta\nu$, the coupling stopband. An estimate of $\Delta\nu$ can be found for the 6.23, 6.20 working point using the following result for the rms $|\Delta\nu|$ due an rms rotation error, $\Delta\theta_{rms}$, in the quadrupoles.

$$\Delta\nu_{rms} = \frac{1}{4\pi} \frac{2\Delta\theta_{rms}}{B\rho} [\sum L^2 G^2 \beta_x \beta_y]^{1/2} \quad (4)$$

where the sum is over the quadrupoles and L is the quadrupole length. For the 6.23, 6.20 lattice this gives

$$\Delta\nu_{rms} = .0042 \tag{5}$$

To estimate the tune splitting, one can use $\Delta\nu = \Delta\nu_{rms} = .0042$. Using this $|\Delta\nu|$, $\nu_1 - \nu_x = -(\nu_2 - \nu_x)$ can be found, giving $\nu_1 - \nu_x = -(\nu_2 - \nu_x) = .00058$. The tune splitting estimate is then

$$\nu_1 - \nu_2 = .0012 + \nu_x - \nu_y \tag{6}$$

The additional tune splitting due to random skew quadrupole field errors is about .0012 .

Local coupling estimate

C_1 and C_2 at any point around can be estimated from the result

$$\begin{aligned} C_1 &= |(\nu_1 - \nu_x)/\Delta\nu|(\beta_y/\beta_x)^{1/2} \\ C_2 &= |(\nu_2 - \nu_y)/\Delta\nu|(\beta_x/\beta_y)^{1/2} \end{aligned} \tag{7}$$

which holds when ν_x, ν_y are near the $\nu_x = \nu_y$ resonance line and no coupling correction has been done. This gives for the 6.23, 6.20 working point with an rms rotation error of 1 mr in the quadrupoles

$$\begin{aligned} C_1 &= .14(\beta_y/\beta_x)^{1/2} \\ C_2 &= .14(\beta_x/\beta_y)^{1/2} \end{aligned} \tag{8}$$

This result may be described as showing 14% coupling between the horizontal and vertical motions. It seems likely that this amount of linear coupling will be small compared to the non linear coupling due to non linear space charge forces and the chromaticity correcting sextupoles.

Vertical dispersion estimate

An estimate of the size of the vertical dispersion generated by the skew quadrupole error fields can be obtained by computing the rms vertical dispersion . Assuming that the vertical dispersion is largely generated by the

rotation error in the quadrupoles , then the rms vertical dispersion, $Y_{p,rms}$, can be computed from the following result for the vertical dispersion Y_p

$$Y_p(s) = \frac{1}{2\sin(\pi\nu_y)} \int ds' (\beta_y(s)\beta_y(s'))^{.5} \frac{2G\Delta\theta X_p(s')}{B\rho} \cos(\pi\nu_y - |\psi_y(s) - \psi_y(s')|) \quad (9)$$

For a rotation error of 1 mr rms in the quadrupoles, one finds

$$Y_{p,rms} = .040m \quad (10)$$

at the vertical quadrupole in the straight section for the 6.23, 6.20 lattice. At $\Delta p/p = .007$, this gives a vertical displacement of .28 mm rms, which is small compared to the available aperture or the full beam size.

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References

- [1] A. Fedotov et al, PAC 2001, Paper RPAH004, (2001)
- [2] A. Fedtov et al, BNL/SNS Tech. Note 091 (2001)
- [3] P.J. Bagley, D.L. Rubin, PAC 1989, p.874(1989)
- [4] G. Bourianoff, F. Pilat, AIP Conf. Proc. 255, p. 299 (1991)
- [5] R. Talman , AIP Conf. Proc. 255, p.66 (1991)
- [6] G. Parzen, BNL Report AD/RHIC-100 (1991) and BNL Report AD/AP-49 (1992)