Longitudinal Coupling Impedance of the SNS rf Cavity

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Addendum to BNL/SNS Technical Note No. 115

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Executive Summary

Measured results for the prototype SNS rf cavity were recorded in the BNL/SNS Technical Note No. 115 and were presented at the September ASAC meeting.[1] Several stray resonances with impedance values reaching $Z_{\|} / n \leq 8\ \Omega$ suggested the potential for beam instabilities. The rf cavity had been measured within its covering box from which it is dc isolated by ceramic rings. These breaks were suspected as the source of the resonances, and then confirmed by measurements. For the present measurements, the ceramic rings were shorted thereby eliminating most resonances and reducing the longitudinal impedance value to $Z_{\|} / n \ll 1\ \Omega$ at frequencies below ~100 MHz. The effect of the rf power supply was simulated by a 1.5 kΩ resistor across the ferrite tuning loop. The shunt impedance to the beam produced by the two-gap cavity is ~ 5.2±0.5 kΩ at the first-harmonic operating frequency. The interpretation of the standard wire measurements with the conventional hp-formula and the log formula is discussed.

Coupling Impedance by Wire Measurements

The longitudinal coupling impedance of a component is conveniently measured by inserting a wire in the center of the beam pipe to form a coaxial transmission line. The forward scattering coefficient s_{21} is measured both for the device under test and a reference tube of the same length. The ratio, $S_{21} = s_{21}^{DUT} / s_{21}^{REF}$, yields the coupling by an appropriate expression, either the conventional hp-formula or the log-formula.[2]

In the typical situation, the characteristic impedance, R_c, of the line is different from the standard impedance of the network analyzer, R_0. Consequently a matching network, such as an impedance transformer, needs to be inserted. Often, for simplicity's sake, a resistive matching is applied. On the input side, forward and backward matching is achieved with a series and parallel resister,

$$R_p = G_p^{-1} = \frac{R_0}{\sqrt{1-\eta}} \sim R_0\left(1+\frac{1}{2}\eta\right)$$

$$R_{in} = R_c \frac{\eta - \left(1-\sqrt{1-\eta}\right)}{1-\sqrt{1-\eta}} \sim R_c\left(1-\frac{1}{2}\eta\right)$$

with $\eta = R_0 / R_c$. Furthermore, on the output side, forward matching is achieved with a series resistor

$$R_{out} = R_c\left(1-\eta\right)$$

As example for the present measurement, the characteristic impedance of the 1.25 mm ∅ wire in the ~15 cm beam tube is

$$R_c = \frac{Z_0}{2\pi} \ln \frac{r_o}{r_i} \approx 288\ \Omega \text{ (vs. 265 } \Omega \text{ measured)}$$

requiring the matching resistors, $R_p \approx 54\ \Omega$, $R_{in} \approx 263\ \Omega$, and $R_{out} \approx 213\ \Omega$.

At sufficiently low frequencies, the component is considered as a lumped element and the forward scattering coefficient follows as

$$s_{21} = \frac{2R_0}{R_0 + \left(R_{in} + R_c + Z_{\|}\right)\left(1 + G_p R_0\right)}$$

The expression for the longitudinal coupling impedance is obtained from the ratio of the scattering coefficients,

$$Z_{\|} = \left(R_c + R_{in} + \frac{R_0}{1 + G_p R_0} \right)\left(\frac{1}{S_{21}} - 1 \right) = 2R_c \left(\frac{1}{S_{21}} - 1 \right)$$

in full analogy to the well known hp-formula.

The numerical results for the coupling impedance are preferably presented as real and imaginary part, with the real part of direct interest to the beam stability analysis,

$$R_{\|} = 2R_c \left(\frac{\Re(S_{21})}{\| S_{21} \|^2} - 1 \right)$$

where $\| S_{21} \|$ is the magnitude of the scattering coefficient ratio. Typically, this ratio is stored in the network analyzer as data/memory. By using the conversion from scattering to impedance format, the instrument can produce directly the real and imaginary part of the coupling impedance.

The measurements in TN 115 were interpreted using the log formula,

$$Z_{\|} = -2R_c \log_e S_{21}$$

This expression is derived as approximation for the case $Z_{\|} \ll R_c$ and is not suited for the wire measurement of the cavity shunt impedance. In contrast to the hp-formula, which is derived for a lumped, localized structure, the log formula sums correctly the small values of separate impedances by taking into account the phase shift between different locations. Apart from the fundamental resonance, the real part of the coupling impedance is given by

$$R_{\|} = -2R_c \log_e \| S_{21} \|$$

with $\| S_{21} \|$ being downloaded in text format from the instrument.

SNS Cavity Coupling Impedance

The coupling impedance of the SNS prototype cavity with the ceramic rings open and no termination at the ferrite loop, i.e. the original condition, is shown in Fig.1 as directly obtained from the network analyzer.

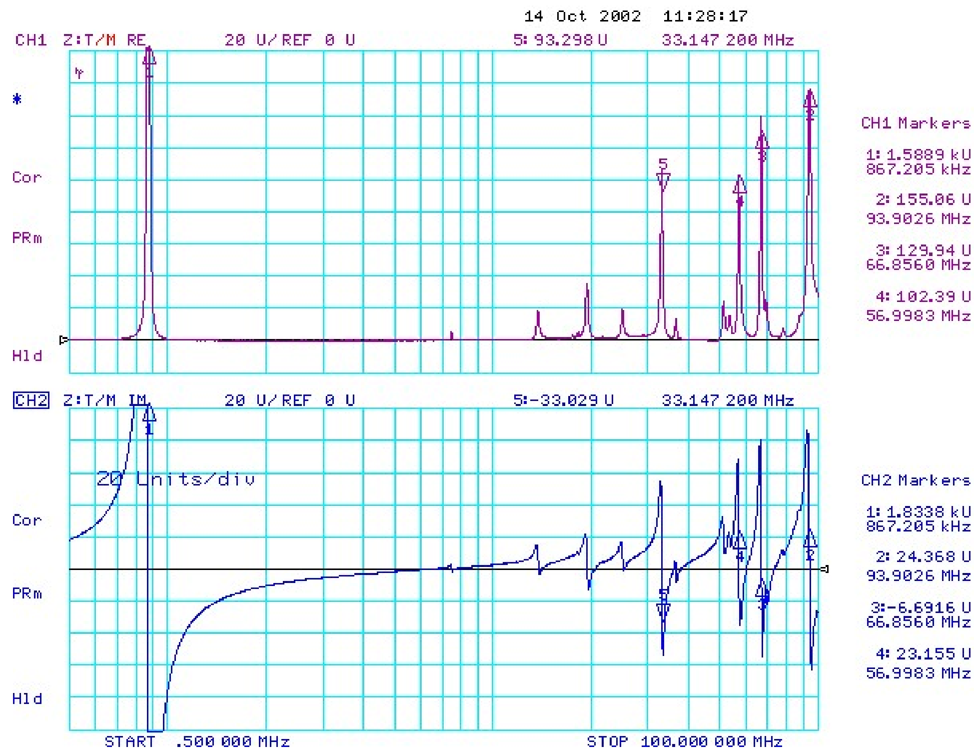

Fig.1. Coupling impedance of SNS cavity with open ceramic rings

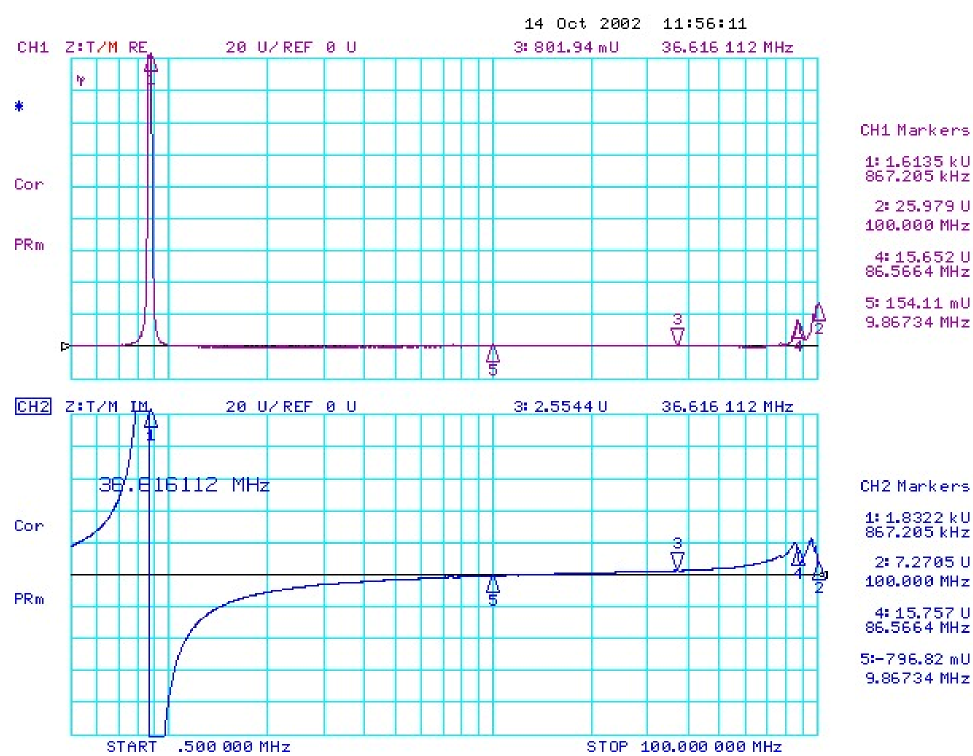

Fig.2. Coupling impedance of SNS cavity with ceramic rings shorted.
(Note that Fig.1 and 2 have the same scale)

Fig 2 shows the cavity with the ceramic rings shorted and the 1.5 kΩ at the ferrite loop. Note that Fig.1 and 2 have the same scale. In the operational cavity, the two ceramic breaks will be shorted by capacitors with 4 μF total.

In Fig. 3, the resistive part of the coupling impedance is shown as obtained from the hp-formula and is compared with the log-formula. The cavity is measured with the ceramic rings shorted and the 1.5 kΩ at the ferrite loop. Using the log-formula in the vicinity of a strong resonance obviously leads to errors. The shunt impedance of the fundamental resonance is 5.2 kΩ calculated from the hp-formula.

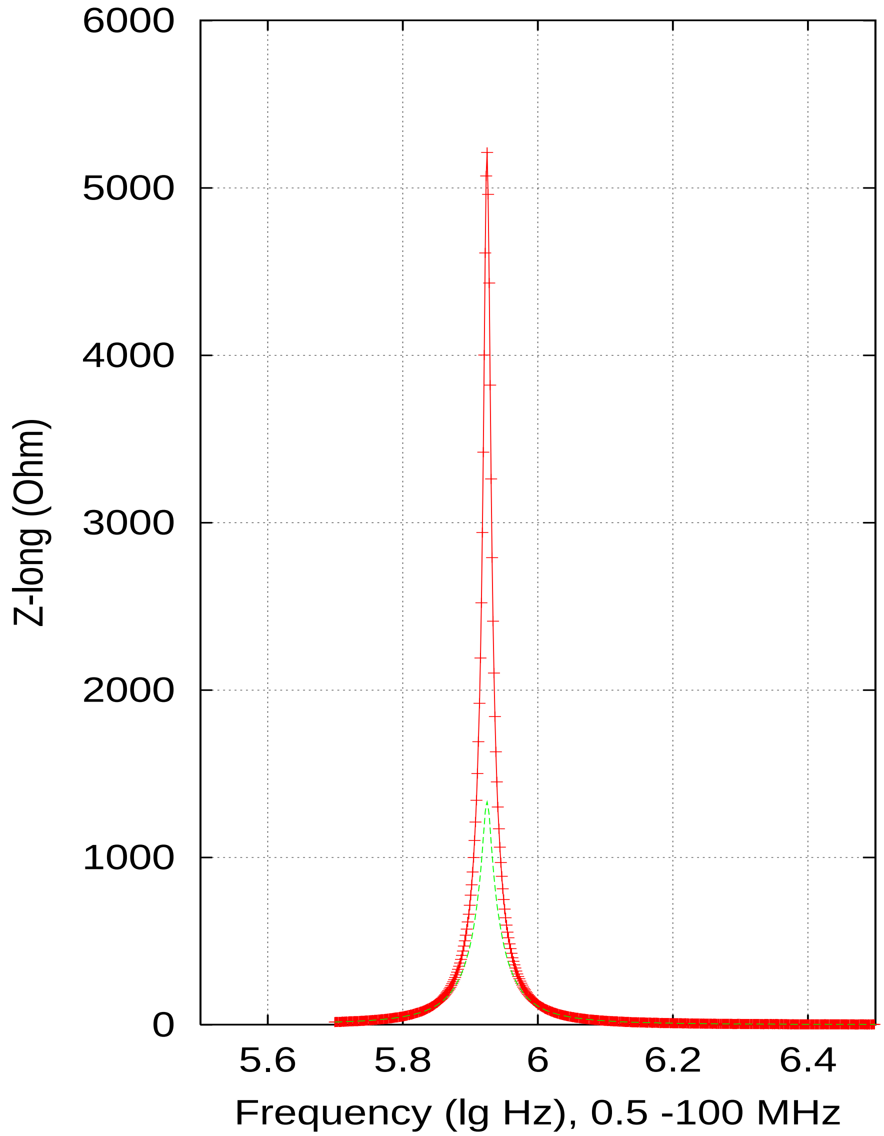

Fig. 3. Impedance at first harmonic from hp (linespoint) and log (lines) formula

An expanded plot of the longitudinal coupling impedance versus frequency is found in Fig. 4.

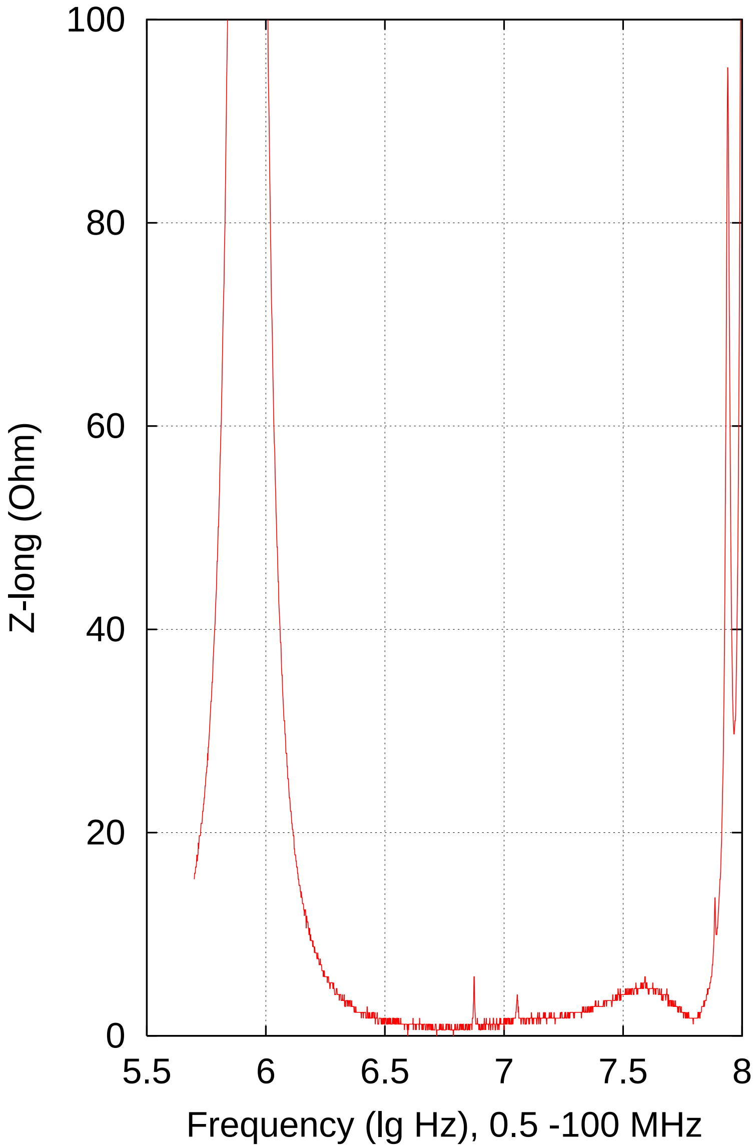

Fig. 4. Expanded view of SNS rf cavity coupling impedance vs. frequency

The remaining small resonances can be parameterized by fitting the measured results to the formal expression

$$Z_{\|} = \sum_i \frac{R_i}{1+\left\{\frac{R_i}{Q_i}\left(\frac{f}{f_i}-\frac{f_i}{f}\right)\right\}^2}$$

The fitting procedure produced the plot for the real impedance part in Fig.5 with the parameters

$f_i(\mathrm{MHz})$	$R_i(\Omega)$	Q_i	$Z_i / n(\Omega)$
7.49	4.5	87.6	0.6
11.37	2.17	58.8	0.2
35.	3.8	1	0.1
87.	74	20	0.9
100	98	19	1.

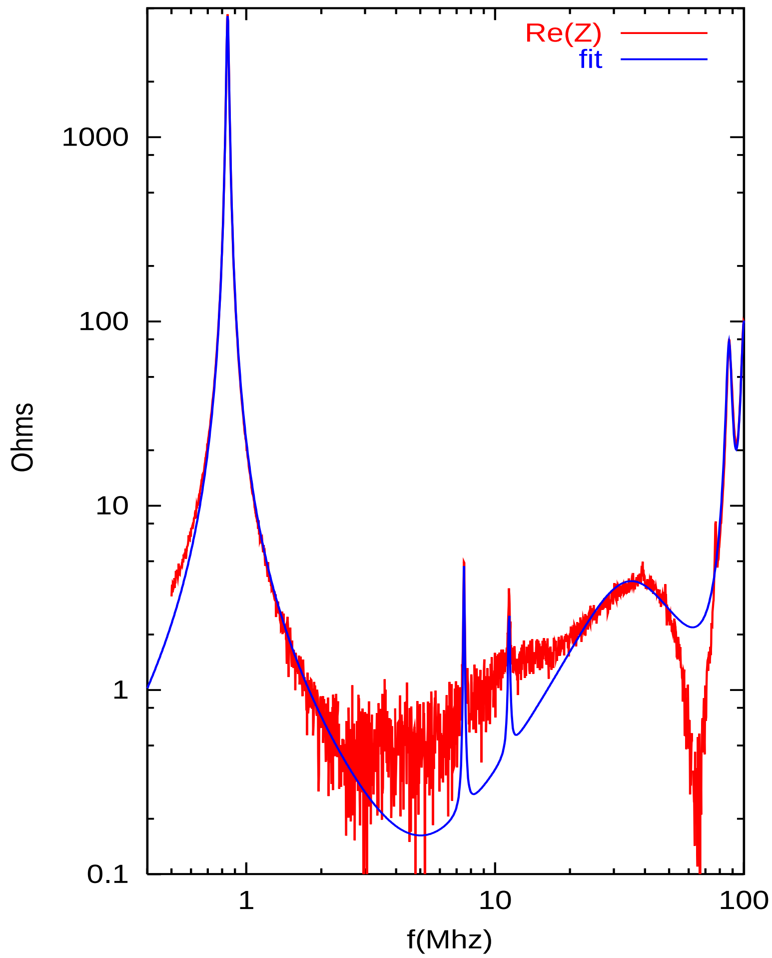

Fig. 5. Formal expression fitted to measured results (note double logarithmic scale)

Acknowledgements

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References

1. D. Davino, H. Hahn, M. Blaskiewicz, *Coupling impedance measurement of the SNS RF cavity,* BNL/SNS Technical Note No. 115 (August 2002)
2. H. Hahn, Phys. Rev. ST Accel. Beams **3**, 122001 (2000)