IMPEDANCE and STABILITY CONSIDERATIONS FOR THE SNS

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Introduction

The Spallation Neutron Source (SNS) is a high intensity machine. For 2 MW operation the average beam current just before extraction is $\bar{I} = 39$ A, with peak currents as high as 95 A. This kind of current, coupled with the loss limit of 0.01%, necessitates a careful investigation of collective effects and instabilities in particular.

Impedance estimates from some sources such as space charge, beam position monitors (BPMs) and the resistivity of the beam pipe are easily calculated with a high degree of confidence. Impedances for other devices, such as kicker magnets, are difficult to calculate with any degree of confidence and should be measured. The present document considers a best guess estimate for these difficult devices.

Given the machine impedance one then needs to estimate the effect this impedance has on the beam. The simplest estimates are those for a coasting beam. For reasonable models of the momentum distribution the instability thesholds and growth rates can be solved for exactly[1]. For bunched beam instabilities the situation is less clear. It is probably true that bunched beam growth rates are are never larger than coasting beam growth rates with the same peak intensity and rms energy spread but, it has never been proven. Additional work is required here.

1 Impedance Estimates

The transverse space charge impedance for a round beam of radius a in a round pipe of radius b is given by

$$Z_\perp = i\frac{CZ_0}{2\pi\beta^2\gamma^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tag{1}$$

where $C = 220$ m is the machine circumference, $Z_0 = 377\ \Omega$, $\gamma = 2.066$ is the Lorentz factor and $\beta = v/c$. For the SNS this impedance is 10 MΩ/m corresponding to a peak current of $I_p = 80$A and a space charge tune shift of $\Delta\nu_{sc} = 0.2$. The longitudinal space charge impedance for the same conditions is

$$Z_\parallel = i\frac{kCZ_0}{2\pi\beta^2\gamma^2}\left(\ln(b/a) + 1/2\right) \tag{2}$$

where $k = 2\pi f/c$ is the wavenumber. For the SNS this impedance is $Z_\parallel \approx i150\Omega n$ where $n = f/f_{rev}$ and $f_{rev} = 1.2$ MHz is the revolution frequency.

The longitudinal impedance due to a matched stripline BPM is given by[2]:

$$Z_{||}(k) = 2Z_s\left(\frac{\phi_0}{2\pi}\right)^2 \left\{\sin^2(k\ell) - i\sin(k\ell)\cos(k\ell)\right\} \tag{3}$$

where Z_s is the characteristic impedance of the matched load, ℓ is twice the length of the stripline and ϕ_0 is the angle subtended by one of the pair of striplines. The transverse impedance for the same device is[3]:

$$Z_\perp(k) = \frac{8Z_s}{kb^2}\left(\frac{\sin(\phi_0/2)}{\pi}\right)^2 \left\{\sin^2(k\ell) - i\sin(k\ell)\cos(k\ell)\right\} \tag{4}$$

where b is the radius of the beam pipe and the transverse impedance is non-zero in the plane the monitor measures. There are 24 BPMs per plane and they are located at maximums of the beta function. The total transverse impedance is $Z_\perp(k) = 24(\sin^2(k\ell) - i\sin(k\ell)\cos(k\ell))/k\ell$ kΩ/m, and $Z_{||}(k) = 180(\sin^2(k\ell) - i\sin(k\ell)\cos(k\ell))\Omega$. Both these expressions are accurate for $k\ell \approx n/200 \lesssim 1$.

For a round beam pipe when the skin depth is small compared to the thickness of the vacuum chamber the longitudinal resistive wall impedance is [4] :

$$Z_{||}(f) = (1 - i\text{sgn}(f))\sum L/2\pi b\sigma\delta \tag{5}$$

where L is the length of a vacuum chamber of radius b, conductivity σ (value for stainless steel), and skin depth $\delta = 1/\sqrt{\mu_0\sigma\pi f}$. For SNS $Z_{||}(f) = 0.74(1 - i\text{sgn}(f))n^{1/2}\Omega$.

The transverse resistive wall impedance is:

$$Z_\perp(f) = cZ_{||}(f)/(\pi f b^2) \tag{6}$$

and is usually the dominant contribution at low frequencies.For SNS $Z_\perp(f) = 5.9(1 - i\text{sgn}(f))n^{-1/2}$ kΩ/m.

Bellows and pipe transitions are more complicated objects and there exist no manageable closed form expressions for their impedances. For low frequencies the longitudinal impedance of a single bellows corrugation or a transition of length ℓ and depth d is given by:

$$Z_{||}(f) \approx -ifd\ell\mu_0/b \tag{7}$$

and the associated transverse impedance satisfies (6). For $d \lesssim b$ and $\pi f b \lesssim c$ (7) is reasonably accurate. Above the cutoff frequency for microwave propagation down the beam pipe a resistive component to the impedance is present. There will also be at least one narrow band resonance below cutoff corresponding to a standing wave. The approximate wavenumbers of these resonances are $k_{||} = 2.4/(d+b)$ and $k_\perp = 3.8/(d+b)$ corresponding to the lowest frequency transverse magnetic waves in a cylinder of radius $= r + b$. For SNS assume that 100m of the beampipe has a 13 cm radius and the rest has a 10 cm radius. Then $Z_\perp = -i360$kΩ/m and $Z_{||} = -i45n\Omega$.

The impedance of the extraction kicker is difficult to calculate. For transverse impedance this device is modeled as a cylindrical sheath of ferrite inside a perfectly conducting pipe. This neglects the shielding effect of the inner conductors and should provide a worst case estimate as long as the driving loop is in series with a large impedance when the kicker is not being fired. Let the inner radius of the ferrite be at $r = b$ and the outer radius be at $r = R$. The radii are chosen so that πb^2 equals the cross sectional area of the vacuum region within the kicker and $R - b$ equals the thickness of the ferrite. Assume the device is long enough so that its impedance is equal to its length

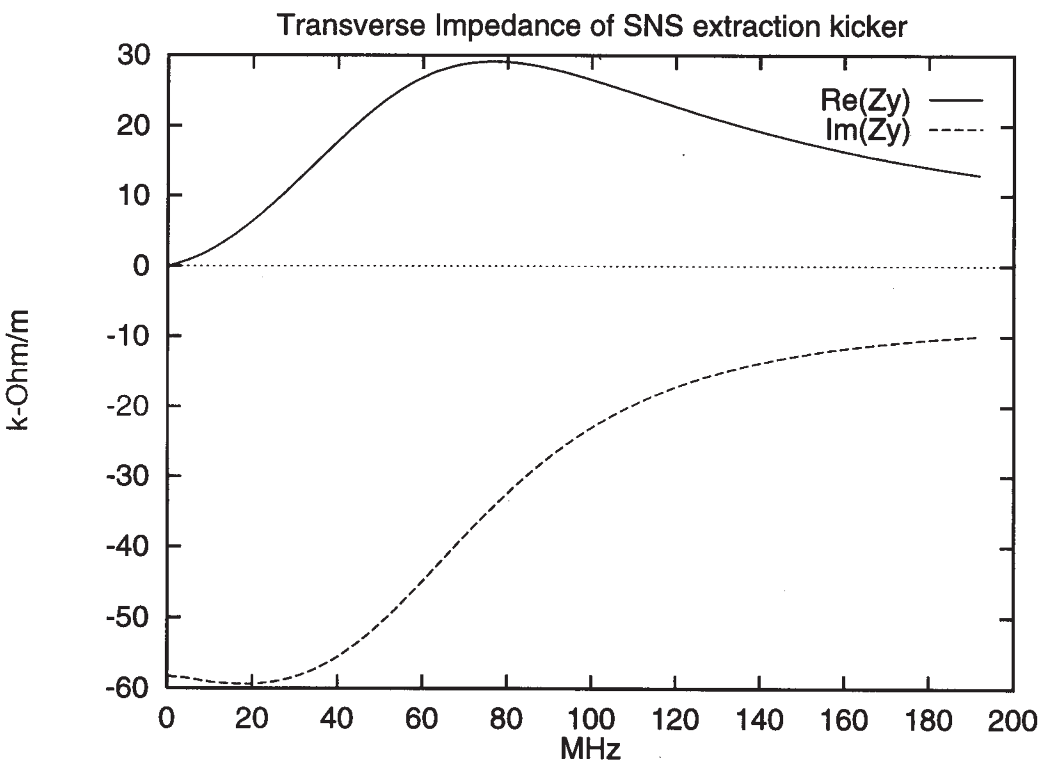
ℓ multiplied by the impedance per unit length of a ferrite lined beampipe. The transverse impedance may be calculated using the techniques describe in Chao [4], in a similar fashion to the resistive wall impedance. The main difference arises from the fact that the outer boundary conditions at $r = R$ need to be handled more carefully. The decay lengths of the fields in the ferrite are usually not small compared to $R - b$. The result is

$$Z_\perp(k) = \frac{-Z_0\ell}{\pi k b^2 \left(ikb^2/2 + (b/\mu)[i/k + \kappa]E'/E - 2\kappa - i/k + \mu b\kappa B'/B\right)}, \tag{8}$$

where $1/\kappa = ik(\mu\epsilon - 1)$, $\mu = \mu(k)$ is the relative permeability of the ferrite, and ϵ is the relative permitivity of the ferrite. The ratio B'/B is the radial derivative of the longitudinal component of the magnetic field divided by its value, evaluated at the ferrite vacuum boundary on the ferrite side. E'/E is the similar ratio for the longitudinal component of the electric field. Both fields obey the equation

$$E'' + E'/r + (k^2(\mu\epsilon - 1) - 1/r^2)E = 0.$$

The boundary conditions at $r = R$ are $E = 0$ and $B' = 0$ which uniquely define the ratios at $r = b$. The ratios can be expressed in terms of Bessel functions [5] but the author has found it easier to numerically integrate the differential equation. For numerical evaluation, measured values of the relative permeability were used for $f < 12$MHz and a power law interpolation was used for higher frequencies. The permitivity was set to $\epsilon = 1$.



The longitudinal impedance of the extraction kicker is seriously overestimated using the model for the transverse calculation. Luckily there are measurements of the longitudinal impedance of a kicker magnet of similar cross section [6]. The measured magnet is 6" long and its longitudinal impedance is well approximated by a resonantor impedance with shunt impedance $R_{sh} = 350\Omega$, resonant frequency

$f_{res} = 60$ MHz and quality factor $Q = 3$. The SNS extraction kicker is made of four magnets and the sum of their lengths is 3.2 m. Taking the ratio of the lengths implies a shunt impedance of $R_{sh} = 7.3$ kΩ for the SNS extraction kicker.

The rf cavities are the final impedance sources considered here. The cavity gaps will be only a few inches long so their transverse impedance should be negligible. The longitudinal impedance of higher order modes will be considered in the early design stages and the final product will have negligible impedance for modes other than the fundamental.

2 Instabilities

For longitudinal instabilities the coasting beam model will be used. For 2MW operation with a dual harmonic system the peak beam current is no more than $I_p = 100$ A and the root mean square energy spread is $\sigma(E) = 4$ MeV. Consider a coasting beam with a rectangular energy distribution. The dispersion relation for a coasting beam longitudinal mode is [7]

$$\frac{\Omega^2}{\omega_0^2 n^2} = \dot{\tau}^2 + i\frac{eI_p\eta}{2\pi E_0\gamma\beta^2}\frac{Z_{\|}(\Omega + n\omega_0)}{n}, \tag{9}$$

where $\eta = -0.193$ is the frequency slip factor, ω_0 is the angular revolution frequency, n is the mode number, $\Omega + n\omega_0$ is the mode frequency, $\beta = v/c$, $E_0 = 1.938$ GeV and $\dot{\tau} = \eta\sqrt{3}\sigma(E)/E_0\beta^2$. For SNS parameters the two terms on the right hand side of (9) are equal in magnitude when $|Z_{\|}/n| = 800\Omega$ which is significantly larger than SNS values. Hence, the square root can be expanded to first order and the growth rate of the mode is $Im(\Omega) \approx 8Re(Z_{\|})\Omega^{-1}\text{s}^{-1}$. For mode numbers $n < 700$, above which impedances begin to drop exponentially, the BPMs and and resistive wall contribute no more than 200Ω, corresponding to a growth rate of $1600s^{-1}$ and do not present any problem. The extraction kicker could be a problem. For a frequency near 60 MHz a coasting beam growth rate of $58000s^{-1}$ is predicted and might result in longitudinal emittance blow up.

For transverse instabilities, a cold coasting beam approximation predicts a grow rate of[7]

$$Im(\Omega) = \frac{qcI_{peak}Re(Z_{\perp})}{4\pi E_0 Q_\beta} \tag{10}$$

where $Q_\beta \approx 5.8$ is the betatron tune. For SNS parameters $Im(\Omega) = 200Re(Z_{\perp}/(\text{k}\Omega\text{m}^{-1}))\text{s}^{-1}$. The most unstable mode from the resistive wall impedance yields a growth rate of 2600s^{-1} which is probably benign. For the extraction kicker the grow rate is 6000s^{-1} which corresponds to 6 e-folding times per millisecond at the peak current of 100 A. If the estimate of the kicker impedance is accurate, this is probably OK as well

3 Conclusions

The main sources of longitudinal and transverse impedance in the SNS have been identified and an estimate of their impact has been made. Beam position monitors should not be a problem. Neither should a stainless steel vacuum chamber. However, the extraction kicker might present difficulties. This device becomes a problem for relatively high frequencies $\sim$ 60MHz, which are difficult to simulate. Additionally, the kicker impedance estimates presented here are rough at best. Measurements at an early stage are clearly in order.

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